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THE EVOLUTION OF SOLAR STARS.[1]

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THE contemplation of our solar system, in which a number of planets move around the Sun in the same direction, and nearly the same plane, gave rise to the idea that our Sun has gradually condensed from a nebulous mass. This idea was confirmed by the discoveries of the telescope, which enabled men to see nebulous masses suspended in the sky, some spread out irregularly, some in spiral forms with denser portions, and some of globular shape, with a starlike nucleus in the center. It was clearly a tempting subject to astronomers and others to speculate on the gradual formation of the stars out of the original nebulous chaos. No wonder, then, that the interest which had been aroused by Kant and Laplace, when they formed their celebrated Nebular Hypothesis, has been growing steadily, especially since the spectroscope gave us the means of studying the material out of which the stars are formed.

The readers of this JOURNAL must all be familiar with the main facts of stellar spectroscopy and the general idea of stellar

[1] Read before the *Royal Philosophical Society of Glasgow*, November 6, 1901. Revised for publication in the ASTROPHYSICAL JOURNAL, January 1903.

evolution which these facts have suggested. Everyone recognizes that some kind of evolution is clearly indicated by the manner in which star spectra classify themselves into groups which, though distinct, are yet connected with each other by intermediate types. But while agreeing on a general process of evolution, there is still a good deal of room for differences of opinion on the life-histories of particular stars. One of the important questions which may be raised is this: Does each star or, at any rate, the great majority of them, pass through each of the stages of a uniform evolution? Has, for instance, our Sun at one time given a spectrum identical with that of *a Leonis?* Further, are we justified in concluding that all stars are made up of the same chemical elements in the same proportion? And lastly, admitting a uniform evolution, what is the meaning, as the star grows older, of the gradual displacement of the hydrogen in its atmosphere, first by calcium, and ultimately by iron and other metals?

Before we enter into a fuller discussion of these points, I shall briefly review the methods of experimental investigation which are at our disposal. The simple spectroscopic analysis, which only tells us of the presence of an element, is now complicated, but improved, by the observed fact that spectra are found to vary according to the experimental conditions. If we volatilize a metal by a powerful spark, it sends out waves which are different from those which are seen with a weak spark; and if we replace the spark by the voltaic arc, which volatilizes more material, but is probably at a lower temperature, or by the oxyhydrogen flame, distinct differences due to the different condition of the luminous vapor generally appear in the spectra. Opinions are not quite concordant as to the cause of these differences, how far they depend on temperature simply, and how far pressure and density may affect them; or whether, finally, the dissociating power of high electric tension may alone be sufficient to explain the observed facts. The metallic absorption in the solar spectrum seems roughly to correspond to that of our electric arc, while, according to Sir Norman Lockyer, the metallic lines seen in the groups of stars intermediate between

the helium and solar spectrum, correspond more closely to the spectra observed in our strongest spark discharges.

In addition to the changes which are produced by temperature and density, we have an effect due to pressure, which consists in a slight lengthening of the waves sent out by the molecules. This effect, which was discovered by Messrs. Mohler and Humphreys, allows us to determine that the pressure to which the vapors in the Sun are subjected is somewhere between two and seven atmospheres.

The investigation of thermal radiation forms another avenue through which we may approach the all-important question of the surface temperature of the stars. It is only quite recently that Mr. E. F. Nichols has succeeded in comparing the total heat radiation of some of the brightest stars. *Vega* and *Arcturus* judged by the eye have the same magnitude, which means that the same amount of that radiation which affects our eyes reaches us from each of these two stars. But when measured by an instrument which is sensitive to all radiation, *Arcturus* is found to have more than double the intensity. As the proportion of total radiation to luminous radiation diminishes with rising temperature, this would indicate a lower temperature for *Arcturus*, and confirm the conclusion, arrived at on other grounds, that the hydrogen stars have a higher surface temperature than the solar stars. Without actual measurements, we may derive the same result from an inspection of the ultra-violet region of the spectrum. This region is made up of rays which are too short to affect our retina, but which produce a photographic effect, and ought to be stronger and more extended, the higher the temperature of the radiating body. We find that in general the hydrogen stars are those which are richest in this ultra-violet light.

These different lines of argument, all leading to the same result, justify us in saying that the surface temperature of the hydrogen stars is higher than that of the solar stars. An extension of the same reasoning leads to the belief that the helium stars have a temperature which is higher still.

The outward appearance and internal constitution of the

stars is not solely defined by the temperature of their surface, and we are in some cases in possession of important information concerning their size, mass, and density. Doppler's principle has received important applications, and will probably yield further results. It might, *e. g.*, give us some indications whether any star is near the point at which instability sets in owing to centrifugal force.

Were a star to revolve around an axis which does not point toward the Earth, and with sufficient velocity to be near the point at which it could throw off a planet or break up into two stars, we could not fail to notice it by the broadening of its lines; but at present there seems little hope that we shall ever witness so interesting an event as the formation of a double star out of one rotating body.[1] Yet there are several cases of double stars where the two bodies must be nearly in contact with each other, and some of these must have been formed, not so very long ago, by the splitting up of a single rotating mass.

Passing on to the light thrown by theoretical investigations on the constitution of stars and other systems, I must notice in the first place those researches which refer to the internal state of gaseous masses condensing under the action of gravitational forces. A mass of gas sufficiently great to collect into a globular body will be denser at the center than near the surface, because the whole weight of the outer layer will cause pressure, and therefore increased density of the central portions. As regards the temperature, we might suppose, in the first place, that there is no difference throughout the mass; but even if this were the case at any one time, it could not long remain so. If the gas be warmer than the surrounding space, radiation will take place, accompanied necessarily by a lowering of temperature in some parts, and consequently the setting up of ascending or descending convection currents. These convection currents will ultimately establish a distribution of temperature which is

[1](Note added February 1903.) Is it possible that the phenomena of so-called new stars may be due to the sudden violent disturbance produced by the formation of a double star? In such a splitting up, gases which are enormously hotter than the surface gases must suddenly be brought to the surface.

not uniform, and has been named by Lord Kelvin the "convective equilibrium."

If a star is a mass of gas in a state of convective equilibrium, its temperature must increase downward. The rate of increase depends to a great extent on the internal constitution of the molecules. At places where the gravitational attraction is the same, the increase of temperature with depth is proportional to the molecular weight multiplied by a number which is 0.4 in the case of gases like mercury vapor, containing one atom in each molecule, and about 0.3 in the case of gases which contain two atoms in a molecule. But as it is exceedingly likely that at the temperature of the stars all molecules are monatomic like mercury vapor, we may base our calculations on that assumption. The investigations of Homer Lane, Ritter, and Lord Kelvin allow us to solve the problem of the distribution of temperature and density within a star, assuming the interior to behave like a perfect gas in a state of convective equilibrium. Lane was the first to see the importance of a conclusion, which may appear paradoxical at first sight, but which is based on strict mathematical reasoning. According to him, a star, while it radiates heat into space, does not cool, but actually becomes hotter, and this is due to the fact that the contraction, which accompanies the loss of heat, is accompanied by an evolution of heat which more than compensates for the loss. We may imagine radiation to take place chiefly from the outside, and there would no doubt be a lowering of the temperature in these outer layers, if all convection currents were artificially stopped. But at the same time the contraction of the outer shell, inclosing the deeper layers, would cause a rise in temperature in the inside, and consequently a disturbance of the thermal equilibrium. This could be re-established only by convection currents, which would supply the lost heat to the outer layers. Calculation shows that the temperature of the center of a star increases in the same proportion as the diameter of a star diminishes; but it must be clearly understood that all these conclusions are based on the supposition that the whole mass of the star behaves like a perfect gas, a supposition which fails to be true at the surface of a star, and

also near its center, but probably holds very nearly in the intermediate layers.

If a star contained only perfect gases, its surface would be formed where the absolute zero of temperature is reached. But the metallic vapors which to a great extent compose the stars condense into a liquid at a temperature of more than 1,000° above the absolute zero, and we must therefore imagine the boundary of the stars to be formed, not at the zero of temperature, but at the place where a cloud-like condensation of vapors takes place, the temperature of these clouds being probably somewhere between 4,000° and 20,000° C. This formation of clouds, though it precludes us from applying Lane's results to the outer layers of stars, does not affect his calculations as to their internal constitution, which probably give us a good representation of the state of a star from the photosphere down to considerable depths. Ultimately, and especially in the case of stars which are already advanced in their condensation, the equations will fail, because when the molecules of a gas become as near to each other as they are in liquids, molecular forces come into play, which prevent the gases from behaving in the ideal manner of a perfect gas. The molecular forces diminish the compressibility, and ultimately the heat which is generated by compression will fail to compensate for the heat lost by radiation. When that period has been reached the star will begin to cool, pass into the liquid state, and soon cease to be luminous.

The questions which meet us when we try to interpret stellar spectra will be more easily understood after we have examined more closely what happens on the surface of the Sun. We may, in the first place, inquire with advantage whether our knowledge of the constitution of that body supplies us any arguments for or against the theory of convective equilibrium, as presented to us by Lane and Ritter. I have calculated, chiefly from the data supplied by Ritter, the density, the pressure, and the temperature of the inside of a star having the same mass and size as the Sun, and behaving like a perfect gas in a state of convective equilibrium. The numbers are given in the accompanying table.

A simple calculation allows us to extend the results to a period of time when the Sun had a larger diameter. The first column of the table defines the position of the different layers in terms of the fraction obtained by dividing the distance of any layer from the center, by the distance of the uppermost layer. The second column gives the density, and the third the pressure in megadynes per square centimeter, that unit being very nearly equal to our atmosphere. The last column shows the temperature, which depends, however, on the nature of the gas; the numbers given apply to hydrogen, assuming it to retain its biatomic constitution, and should in other cases be multiplied by the molecular weight. Thus assuming the whole of the inside of the Sun to be made up of hydrogen split up into monatomic elements, the temperature at the center would be 12,000,000 degrees. If made up entirely of monatomic iron, the temperature would have fifty-six times that value. To apply the table to a previous period in which the diameter of the Sun was, *e.g.*, twice as great, we should have to divide the second column by 8, which is the cube of 2, the third column by 16, which is the fourth power of 2, and our fourth column by 2. It is a curious fact, which is not perhaps without significance, that the central density of the Sun, as calculated on the assumption of its being a perfect gas, is only very little in excess of the density which, according to the most careful recent estimate, is to be ascribed to the central portion of the Earth, and again that that estimate is very little in excess of the density of solid iron.

GASEOUS SPHERE IN CONVECTIVE EQUILIBRIUM.

Total mass = mass of Sun = 2×10^{33} grams.
a = molecular weight compared to hydrogen.
R = radius of sphere.
= 700,000/ β kilometers,
where β is a factor which is equal to unity in the case of the Sun.
r = distance from center.
$x = r / R$.
Mean density = $1.406 / \beta^3$.

x	$\beta^3 \times$ Density	$\beta^4 \times$ Pressure in Dynes per Square Centimeter	Temperature
0	8.44	8.65×10^{15}	$24.60 \times 10^6 \times \alpha / \beta$
0.1	8.17	8.19	24.06
0.2	7.39	6.93	22.51
0.3	6.23	5.21	20.07
0.4	4.88	3.47	17.07
0.5	3.54	2.03	13.77
0.6	2.33	1.01	10.43
0.7	1.36	0.41	7.28
0.8	0.65	0.12	4.45
0.9	0.20	0.017	2.02
1.0	0.00	0.00	0.00

If it is allowable to imagine the Sun to consist chiefly of iron vapor, that vapor, when above the so-called critical temperature, might be expected to follow the laws of a gaseous compressibility until the density is nearly equal to that of liquid iron, and, whatever the temperature, we shall not be able to compress gaseous iron to a density greater than that of solid iron. It is also allowable to conclude from this reasoning that the distribution of density in the interior of the Sun is not very much different from that indicated in the table, being probably rather less in the central portions, and hence rather greater in the outer portions of the solar mass.

Though the distribution of density and pressure in the interior of a star is probably fairly well represented by the table, the temperature almost certainly is considerably less through the greater portion of the mass. The failure of our equations in this respect is due to diminished compressibility, and also to our having left all effects of radiation and conduction of heat out of consideration. The effects of conduction will be most marked where, owing to increased density and diminished gravitational attraction, the convection current becomes less effective, as for instance near the center of the star. The effects of radiation will be most marked near the surface, and especially in those portions of the star which lie above the cloudy condensations which we have considered to make up its surface. Gases like hydrogen will reach to some height above the surface, and form an atmosphere around the main body of the star. Were it

allowable to neglect radiation altogether, these gases would be in convective equilibrium and rapidly diminish in temperature as they rise above the surface; they could not in fact rise much above the surface before they would be reduced to the zero of temperature.[1] The fall in temperature with altitude above the surface of the Sun is, according to calculation, $26 \times a$ degrees centigrade per kilometer, where a is, as before, the molecular weight. Thus for monatomic iron vapor the diminution in temperature would be 73,000° C. for each 100 kilometers.

It may not be unnecessary to say a few words as to the evidence we possess that the convection currents which play so important a part in the theoretical investigation actually exist in the Sun. The surface radiates an amount of heat into space of which we can form a very fair estimate by measuring the quantity which reaches the Earth. The number so obtained is 1,340,000,000 calories per square meter of the solar surface, the unit of heat being the amount necessary to raise one gram of water through one degree. We obtain an idea of what that number means if we imagine the Sun to be surrounded by a shell of ice; the heat supplied by radiation could melt in each minute a layer of ice fifty-eight feet thick. Or, expressing it with Lord Kelvin in terms of power, we may say that the solar surface does work by radiation equivalent to 131,000 horse-power for each square meter of his surface. The heat thus lost by radiation must be supplied from the inside of the Sun, otherwise the solar surface would cool down in a fraction of a second to a temperature at which it would cease to be luminous. If the heat is carried from the inside to the outside by convection alone, the velocity of the currents of vapor must be very great. Taking the pressure of the vapor near the surface to be one atmosphere, we may say that all the heat contained in a layer having a thick-

[1] (Note added February 1903.) Unless radiation can by itself alone establish a state of equilibrium, convection currents must still take place, and be the predominant factor in the distribution of temperature. Professor Sampson has tried to establish a state of temperature for the case of radiation alone, but his distribution is unstable. As far as my present results go, radiation cannot seriously affect the temperature distribution in the inside of the Sun, unless the material composing it is very much more transparent than we have a right to expect.

ness of 370 meters is lost by radiation in each second of time, and this number does not depend on the nature of the vapor or on its temperature. A layer of that thickness would have to be replaced by convection in every second if the temperature of the surface is to be maintained. From this I calculate that if the difference in pressure between the descending and ascending currents is one atmosphere, the velocity of the convection currents must be 616 meters per second, or about 1,000 miles per hour. These up-and-down draughts of vapor must take place with the calculated velocity, unless an appreciable portion of the heat is supplied from the inside in some other way, as for instance, by radiation. It is difficult to form an estimate as to how far radiation can help to keep up the temperature of the surface. I have made some calculations on that point which, though they have yielded interesting results, cannot at present be expressed in definite numbers. It is sufficient for the present argument to maintain that, even if radiation takes a prominent part in the determination of the distribution of temperature, we cannot escape the conclusion that convection currents must bring about a continuous interchange of matter between the inside and outside of the Sun. This theoretical conclusion is amply confirmed by observation, as is shown by the violent motion observed in the chromosphere and prominences.

In the solar eclipse which was observed in the West Indies in the year 1896 one of these prominences reached to a distance of 140,000 miles. This is by no means the greatest height that has been observed, a prominence being photographed in 1895 by means of an ingenious method due to Professor Hale, which reached to a distance of 281,000 miles from the Sun's limb. The rapidity with which these prominences appear to rise and change their appearance is not perhaps a conclusive proof that the gases which they contain move with great velocity, for these gases are quite possibly always present, and the prominence may only be a sudden lighting up of the gas, or the rapid transmission of an effect, which does not actually require the transmission of a material body. But the tangential velocities observed at the limb of the Sun within the chromosphere have not at present been

explained in a satisfactory way except by assuming that there is an actual motion of hydrogen and of the other gases which form the chromosphere. This tangential motion is far more violent than anything which is required for the convection currents necessary to maintain the temperature of the solar surface. Professor C. A. Young, of Princeton University, states that a velocity of a hundred miles a second is often exceeded, and that twice this velocity is occasionally reached. There can hardly be a doubt as to the facts, but their explanation seems to me more difficult than has been generally recognized. Professor Young says on this point:[1]

> It would seem that thus we might explain how the upper surface of the hydrogen atmosphere is tormented by the uprush from below, and how gaseous masses thrown up from beneath should, in the prominences, present the appearances which have been described. Nor would it be strange if veritable explosions should occur in the quasi pipes or channels through which the vapors rise when, under the varying circumstances of pressure and temperature, the mingled gases reach their point of combination; explosions which should fairly account for such phenomena as those represented in Figs. 69 and 70, where clouds of hydrogen when thrown to an elevation of more than 200,000 miles with a velocity which must have exceeded, at first, 200 miles per second, and very probably taking into account the resistance of the solar atmosphere, may, as Mr. Proctor has shown, have exceeded 500—a velocity sufficient to hurl a dense material entirely clear of the Sun's attraction, and send it out into space, never to return.

My doubt as to the correctness of the above explanation is based on the fact that the highest velocity that a gas can reach when forced to move by differences of pressure is equal to the velocity of sound in the gas, where, of course, the temperature of the gas has to be taken into account in calculating the speed of sound waves. In order that a velocity of 100 miles a second may be possible in monatomic hydrogen, it is necessary that the temperature of the gas should be more than two million degrees, for in no other way can the velocity of sound in hydrogen reach so high a value.[2] On the other hand, if at a temperature of

[1] *The Sun*, p. 207.

[2] The velocity of sound may for violent disturbances be greater than that calculated by the ordinary formula, but there is reason to believe that the calculated velocity cannot be exceeded many times.

(Note added February 1903.) Since the above was written, Professor Julius, of

10,000 C., at which we may imagine the solar surface to be, a gas can rush out under the action of a pressure, however great, with a speed of 100 miles a second, the mass of its molecules must be over 200 times less than the mass of the hydrogen atom such as we know it. Sir Norman Lockyer has recently expressed the opinion[1] that the true spectrum of hydrogen such as is seen in the prominences is due, not to the hydrogen atom as we know it, but to a much smaller one, derived from it by dissociation, and he has estimated this smaller atom to have a mass about sixty times smaller than the hydrogen atom. If this view were accepted the difficulty would disappear, and the observed high velocities might be explained by internal pressure. But there is a difficulty in believing such small masses to be capable of emitting visible radiations.

Leaving out of account for the present the possibility of such a great subdivision of atoms, there are, to my mind, only three courses open to us. We may, in the first place, deny the necessity of admitting the existence of velocities as great as those I have named. As regards radial motion outward from the Sun, the evidence in favor of the reality of these velocities is not perhaps conclusive.[2] But the tangential velocities at the limb of the

Utrecht, has made the ingenious suggestion that many of the appearances we observe on the Sun's limb are not real, but are due to an optical illusion, produced by anomalous dispersion. I do not think that, so far, the efforts to account for prominences in this way have been successful, but at present I only wish to point out that the explanation does not get rid of the difficulty which has been pointed out in the text, for the rapid change in appearance, which indicates apparently a large radial velocity, would, according to Professor Julius, still be accounted for by the propagation of some disturbance, though not by a projection of matter. But the velocity of sound is the limiting velocity for the propagation of any disturbance, whether of matter itself or of any arrangement of matter, and, therefore, a large observed velocity is equally fatal to the theory of anomalous dispersion and to the older theory.

[1] *Inorganic Evolution*, p. 182.

[2] (Note added February 1903.) Professor Hale (*Astronomy and Astro-Physics*, **2**, 611 and 917) has described a remarkable outburst on the Sun's surface, the vapors produced by the outburst quickly covering a small portion of the Sun's disk, and obscuring two Sun-spots and a number of faculæ.

After the outburst, however, it was found that no permanent change in the appearance of Sun-spots and faculæ had taken place, and Professor Hale concludes from this that the obscuring vapors must therefore have been well above the layer of spots and faculæ. The velocities of those vapors during the outburst must have been very large indeed.

Sun, which are velocities parallel to the solar surface, have been observed to be as great as the radial velocities, and these must be real unless some other cause may produce displacements of spectroscopic lines. One such cause recently discovered, and already mentioned, is pressure, which increases the wave-length. Other effects might be thought of. which may act in the same direction, but it is much more difficult to imagine any cause which can produce a *shortening* of wave-length. As far as I can judge from the published accounts and drawings, a great tangential velocity at the Sun's limb is as often observed to take place *toward* us as *away* from us; hence, even if two causes may act, one shortening and the other lengthening the waves, it does not seem probable that the two causes would act with equal frequency and to an equal extent in both directions. For the present we are forced to admit only known effects, and hence we are forced to recognize the reality of the great velocities which have been deduced from the observations.

Various attempts have been made to account for a number of phenomena which are observed on the solar surface by electrical actions, such attempts being based on the supposition that the Sun as a whole is a highly charged electrified body. But we know that no body at the temperature of the solar surface could permanently retain an electric charge. If there is therefore a permanent electric force, there must also be a permanent electromotive force tending to drive negative electricity from the inside to the outside, or *vice versa*. There is nothing improbable in such a supposition, as the phenomena of atmospheric electricity show. The surface of the Earth is charged with negative electricity, and though we know that every burning fire, and every wave of the sea breaking into spray, tends to dissipate that charge, it yet is permanent, and remains without apparent diminution. We conclude that some cause, upon which meteorologists and physicists are not yet agreed, exists, which tends to bring the dissipated electricity back to the Earth, and we are confirmed in this conclusion by the fact that falling drops of rain are more frequently charged negatively than positively.

We are therefore quite at liberty to admit a high charge of

electricity at the surface of the Sun, which probably does not, however, exert any electric force, except near its surface; the outside being screened by opposite electrification. The only way which occurs to me as possibly causing an appreciable electric force at a distance greater than the solar diameter would be to suppose the existence of highly eccentric meteoric swarms, which, passing near the Sun, would carry the neutralizing charge out into space. The streamers of the solar corona might be due to electric discharges between such retreating swarms and the Sun.

If the Sun is a highly charged electrified body, velocities like those observed in the atmosphere are possible, if these velocities are radial, *i. e.*, from the center of the Sun or toward it, but I am unable to satisfy myself that the observed tangential velocities can be explained by electric action. There remains only one way of accounting for these velocities, and that is to conclude that they are either directly due to meteoric matter circulating around the Sun, or indirectly to meteoric matter falling into the Sun, and locally generating a temperature sufficiently high to allow of molecular velocities of 200 miles per second. The velocity of a piece of matter circulating around the Sun and close to its surface in a circular orbit is about 270 miles per second. Such a piece of matter would tend to carry with it the very tenuous gases which are floating above the solar surface, and may well impart to them a velocity ranging from 100 to 200 miles. If actually falling into the Sun, the conversion of their motion into heat, or the kinetic energy supplied by the splash, would be sufficient to account for the observed velocities.[1]

The explanation of the violent disturbances which are observed to take place on the solar surface does not come within the range of my main subject, but it was necessary to point out

[1] (Note added February 1903.) Matter falling into the Sun and producing what has been called a "splash" could account for velocities very much larger than that of the velocity of sound; in fact, there is no limit to the velocity that could be generated in this fashion, as the energy of impact is, at the first instant, concentrated into a very small amount of matter. If the splash takes place near the Sun's limb, but on the visible portion, great receding tangential velocities may be observed. If the splash takes place behind the Sun's limb, the tangential velocities would be approaching the Earth.

that the convection currents, which are necessary to the temperature distribution which is now generally admitted by astronomers, are actually observed to take place on the surface of the Sun with greater violence than we might *a priori* have expected. A detailed study of solar phenomena is in my opinion the only sure guide in our investigations on stellar constitution, and it is probable that many of the unsolved problems, which still meet us in the interpretation of stellar spectra, will be cleared up in the solar and terrestrial laboratories rather than by mere statistical comparisons and classifications. There are, in fact, many similarities between solar and stellar phenomena. One of the most curious facts revealed by the photographs of star spectra is, as has already been pointed out, the peculiar behavior of calcium, which forms the main connecting link between hydrogen and metallic stars; and this behavior of calcium shows itself with equal persistency in the spectra of solar prominences, which, as regards the hydrogen lines, do not present a spectrum far different from that of *a Aquilæ* or *Procyon;* while the similarity between the spectrum of the chromosphere and that of some of the stars has been pointed out by Sir Norman Lockyer.

Before leaving the subject of the Sun, I may refer briefly to an argument which seems to me to be fatal to any theory which involves the decomposition of the elements right through its mass. We may say with certainty that no amount of pressure can increase the density of any substance much beyond what it is in its liquid state. Liquid hydrogen has a density of about 0.09, or less than the fifteenth part of the average density of the Sun. I conclude that the interior of the Sun cannot be made up of hydrogen or of any substance which might be formed by the breaking up of hydrogen, for such decompositions are, as far as we can judge, never accompanied by an increase of density. On the other hand, the mean density of the Sun is quite consistent with the supposition that its interior is mainly composed of the same substances as are known to us on the Earth.

We may now return to the main subject of our inquiry, which is the critical discussion of the arguments that have convinced the great majority of astronomers of a process of evolution

which in the course of time makes each star pass successively through a number of stages, in which the spectrum changes from that of the helium stars to that of the hydrogen stars, and hence to that of stars with prominent calcium lines and of the solar stars.

The first fact which requires explanation is the one which is generally, though not universally admitted, that the temperature of the photosphere of the stars diminishes in the order in which they have just been named. If the hydrogen star is always hotter than the solar star, this would suggest that the chemical composition stands in some direct causal relationship to the temperature, but it is open to discussion which is the cause and which the effect. Is the photosphere of *α Leonis* hotter because it is surrounded by an atmosphere chiefly containing hydrogen, or does *α Leonis* only show us the hydrogen spectrum because its atmosphere is too hot to show anything else? The discussion of this point is altogether independent of our ideas regarding evolution. Even if we do not wish to enter at all into the previous history of a star or its future development, we are bound to search for an explanation of the constitution of the present universe, which shows us hydrogen stars and solar stars, the former being apparently hotter than the latter.

But if we take the theory of evolution into account, we have further to explain the fact that, if the above be true, a star cools as it grows older, while the theory of Homer Lane, of which an outline has been given, states that the star should get hotter. The apparent disagreement between theory and observation has been a stumbling-block to astronomers, but it is due in great measure to the want of definiteness in our meaning, when we speak of the "temperature" of a star. We do not observe the temperature of the center, or the temperature of the gaseous mass below the photosphere, and it is with this temperature that the theoretical analysis deals. What we can observe is the photosphere and the absorbing layer above it, and the temperature of these portions of the Sun are not touched by Lane's theory. If our ideas of the photosphere are correct, and it consists of condensed clouds of metallic or carbon

vapor, the temperature of these clouds will be quite independent of the temperature inside the star; and, for all we know, might under certain circumstances remain the same for a long period of a star's life, during which time the star may condense to a fraction of its original volume, and its interior become hotter and hotter, until the condensation has reached the point at which the laws of gaseous compression no longer hold. The surface of a star is pouring out energy in the form of radiation, and the temperature of the surface will depend on the balance of a number of delicately poised conditions. Equilibrium is reached when the loss of heat by radiation is balanced by an equal gain of the heat from the inside or from the outside. The gain in the case of our Sun is in great part due to the convection currents from beneath, which keep the photosphere at the temperature at which, under the existing pressure, the metallic vapors condense. So far we should expect the photosphere to become more luminous as the star contracts, because the greater the intensity of gravitational attraction, the more active the convection currents may be expected to be. But two factors may operate in the opposite direction. In the first place, we must not assume that the inflow of outside meteoric matter, which not so long ago was considered to be the chief cause of the maintenance of solar heat, is altogether inactive. Even in the case of the Sun, it has already been pointed out that several phenomena point directly to the generation of heat at the surface of the Sun by the impact of falling masses. What in the Sun is a subordinate cause may in some of the stars become predominant, and the photosphere may recuperate itself for the loss of energy which it radiates into space, not only from the inside, but also from the outside. If this is the case, we need not be surprised that the temperature of the photosphere is apparently higher in the younger stars, or that the spectrum of these younger stars resembles that of the solar prominences.

But even without having recourse to outside influence, it is possible to account for the higher temperature of the hydrogen stars in a more direct way, by making the hydrogen atmos-

phere itself responsible for it. The temperature of the photosphere must be largely affected by the absorbing properties of the gases surrounding it. If, for instance, these gases were largely to absorb the infra-red radiation, the effect of such absorption would be observed in a rise of temperature. We need only point, in illustration of this, to the way in which the glass roof and sides of a hothouse protect the plants inside against loss of heat by radiation into space. If it were possible to imagine hydrogen to absorb infra-red rays, this gas would, by stopping the loss of these rays, increase the visible, and especially the blue and violet, radiations, and the fact that the metallic lines shown by hydrogen stars are principally the high temperature lines would thus be accounted for. Such an explanation will only remain a mere surmise, unless it is confirmed by independent evidence, but perhaps the phenomena accompanying the formation of faculæ on the Sun may be found to furnish such evidence, although only of an indirect character. The faculæ are bright streaks on the solar surface specially seen in the neighborhood of spots, and, according to recent observations, they are closely connected with the prominences which seem chiefly to lie above them. If the suggested explanation for the high temperature of hydrogen stars be correct, the same explanation would apply to any portion of the solar surface which has masses of hydrogen hanging over it, and the increased luminosity of those portions of the Sun which lie underneath the prominences, would be a necessary consequence. To prevent misunderstanding, it is well to point out that the infra-red absorption need not be due to the same molecules of hydrogen which give the well-known hydrogen spectrum, and which are almost certainly not identical with the diatomic hydrogen molecule which we prepare in the laboratory. The luminous hydrogen in the stars must be surrounded by cooler hydrogen, and the space surrounding the prominences will similarly contain masses of cool hydrogen, having radiating and absorbing properties differing from those of the luminous substance. If I have dwelt on an explanation which at present is little more than a guess, it is only to emphasize that we need

not hesitate on theoretical grounds to accept the evidence of observation, that the photosphere of the hydrogen stars is hotter than the photosphere of a star giving a solar spectrum. The suggestions I have put forward are sufficient to show that the temperature of the photosphere is regulated by considerations which lie altogether outside the calculations of Lane.

We are now prepared to admit that the hydrogen star is hotter than the solar star, and that, if there has been a process of evolution, this hydrogen period of a star is earlier than the solar period. The reason for this second statement will appear more clearly farther on. This brings us to the next stage of our problem. Why does the hydrogen disappear in the process of cooling, and why is its spectrum replaced by that of the metallic vapors? The simplest explanation — simple because it cuts the Gordian knot — is that offered by Sir Norman Lockyer. If the hottest star shows no metallic lines, it is because the temperature is too high for the existence of the molecule which alone can emit the radiation corresponding to these lines. The atoms are decomposed or dissociated, or whatever name we may attach to what must practically be a splitting of what is generally considered an "atom," or, in other words, unsplitable by chemical agencies. When a star cools, and its photosphere has reached the temperature at which the more complex molecule can exist, the corpuscles, according to Lockyer's theory, will recombine and ultimately form metallic vapors such as we know on the Earth. This explanation involves a hypothesis that is possible and consistent, but which, before it can be generally accepted, must either be shown to be the only hypothesis consistent with the facts, or to be supported by strong outside evidence. A great difficulty of the dissociation hypothesis lies in the fact that, as in the case of the Sun, so also in the case of the *Algol* variables, of which the density is approximately known, that density is greater than the density of solid hydrogen, although these stars have spectra which are generally of the hydrogen or calcium type. In order to maintain the theory, it would be necessary to imagine that dissociation only takes place on the surface of the star, and that the pressure inside is sufficient to produce recombination, in spite of the higher temperature which reigns there.

What are the alternative suggestions which have been made? Sir William Huggins, who touched on this in his presidential address delivered to the British Association at Cardiff, draws attention to the effect of convection currents in mixing up different layers of the gaseous matter forming the star. If convection currents could be completely stopped, the heavier gases would sink to lower levels, and the outer layer of a star would be made up of hydrogen and the lighter metallic vapors. It is owing to convection that a mixing takes place, and the stronger the convection the more complete is this mixing. The following quotation will show the position Sir William Huggins takes up in this matter:

> Now, the conditions of the radiating photosphere and those of the gases above it, on which the character of the spectrum of a star depends, will be determined, not alone by temperature, but also by the force of gravity in these regions; this force will be fixed by the star's mass and its stage of condensation, and will become greater as the star continues to condense.
>
> In the case of the Sun the force of gravity has already become so great at the surface that the decrease of the density of the gases must be extremely rapid, passing in the space of a few miles from the atmospheric pressure to a density infinitesimally small; consequently the temperature-gradient at the surface, if determined solely by expansion, must be extremely rapid. The gases here, however, are exposed to the fierce radiation of the Sun, and, unless wholly transparent, would take up heat, especially if any solid or liquid particles were present from condensation or convection currents.
>
> From these causes, within a very small extent of space at the surface of the Sun, all bodies with which we are acquainted should fall to a condition in which the extremely tenuous gas could no longer give a visible spectrum.
>
> * * * * * * * * * * * * * *
>
> Passing backward in the star's life, we should find a gradual weakening of gravity at the surface, a reduction of the temperature-gradient as far as it was determined by expansion, and convection currents of less violence producing less interference with the proportional quantities of gases due to their vapor densities, while the effects of eruptions would be more extensive.
>
> At last we might come to a state of things in which, if the star were hot enough, only hydrogen might be sufficiently cool relatively to the radiation behind to produce a strong absorption. The lower vapors would be protected, and might continue to be relatively too hot for their lines to appear very dark upon the continuous spectrum; besides, their lines might be possibly to some extent effaced by the coming in under such conditions in the vapors themselves of a continuous spectrum.

In such a star the light radiated toward the upper part of the atmosphere may have come from portions lower down of the atmosphere itself, or at least from parts not greatly hotter. There may be no such great difference of temperature of the low and less low portions of the star's atmosphere as to make the darkening effect of absorption of the protected metallic vapors to prevail over the illuminating effect of their emission.

In a discussion before the Royal Society in 1897 I took up an essentially similar position.[1] "The chief difference (I then wrote) between a hydrogen and a solar star lies in the more or less effectual mixing up of the constituents. If we could introduce a stirrer into *a Lyræ* there can be no doubt whatever that the low-temperature lines of iron would make their appearance, while, on the other hand, if we could stop all convection currents on the surface of the Sun, the hydrogen which now lies under the photosphere would gradually diffuse out and give greater prominence to its characteristic absorption."

I still believe this statement to be true, but I have modified my opinion in so far as I do not now believe the difference in the condition of the surface of stars like *Sirius* or our Sun to be sufficient to eliminate convection almost entirely in one case and make it the predominant factor in the other. The mean density of *Sirius* is probably not greater than that which our Sun had when its diameter was about five times as great as it is now. But even then, with a gravitational force still greater than that on the surface of the Earth, convection currents must have been active in stirring up and mixing the strata down to a considerable depth below the surface. The conditions which I imagined, in 1897, to hold in the hydrogen stars are not, according to my present opinion, consistent with the formation of a photosphere, which, I now believe, necessarily involves effective convection currents. Hence, there must be some other cause for the elimination of hydrogen out of the atmosphere of stars, when they have reached the solar stage. We are reasoning, of course, on the supposition that the difference in the type of spectra is not due to any inherent chemical difference in the composition of the stars. The evidence for this assumption will have to be further examined, especially in view of the fact that, if my suggestion

[1] *Proc. R. S.*, **61**, 209.

should prove to have a solid foundation, and if the presence of masses of hydrogen is the cause and not the result of the higher temperature of the photosphere, the difficulty we are now trying to overcome disappears. Here, as in the previous discussion, it is not so much my intention to argue in favor of one hypothesis or another, but rather to show the different possibilities which may, in a natural way, account for much that is obscure at present.

If we had only to account for those stars which chiefly show hydrogen and calcium, we might attribute their spectra to the action, in an exaggerated form, of the same cause which produces the prominence spectrum of our Sun. And if we believe that the influx of meteoric matter is directly or indirectly responsible for the prominences, we should be led to suppose that the stars of the *Procyon* type are bodies in which the aggregation of meteoric masses from outside still plays an important part in regulating the temperature and spectrum of the superficial layers. This, though in many ways a satisfactory explanation, does not account for the spectra in which hydrogen is seen without the calcium, nor for other types of spectra which are generally considered to precede the *Procyon* type.

But the disappearance of hydrogen as the star condenses is not perhaps a phenomenon which should surprise us so much. Hydrogen is readily absorbed by many metals, even at a high temperature, and it is highly probable that gases show phenomena of molecular absorption like solids or liquids, when they are subjected to a pressure so high that their density approaches the density of the liquid state. I can see nothing improbable in the supposition that when a star condenses, and its pressure reaches a high value in the interior, it should begin to absorb hydrogen, helium, and possibly oxygen, nitrogen, and the other constituents which have either not been observed in the Sun, or only give faint evidence of their presence. If this opinion is correct, a quantity of matter, suddenly transported from the interior of the Sun to the outside, would violently give up the hydrogen which it was able to contain under its original pressure. The phenomena observed on the surface of the Sun would seem

to lend countenance to such a view; at any rate they do not contradict it. Another and perhaps simpler explanation is suggested by considering the process of the formation of a star from its first beginnings. Its consideration may therefore be deferred until we are prepared to look at the question of evolution as a whole. I repeat that it is only my intention to put forward suggestions, which may be found to have some truth in them, and which should, therefore, be taken into consideration.

But from such speculative inquiries we may once more turn to the more solid search for further facts. Results which have an important bearing on our subject have been obtained from the observation of double stars, for they allow us to obtain a value for the state of condensation of the matter composing some of these stars. Professor E. C. Pickering and, later, Mr. Monck have deduced a remarkable equation, which connects together the intrinsic brightness of a star's surface, its mean density, and other quantities which may be obtained by observation. We may thus calculate the density on the supposition that all stars emit an equal amount of light per unit surface. As regards stars showing a similar type of spectrum, this supposition is probably nearly correct, but the results have to be used with caution when comparing a hydrogen and a solar star, for the mere fact that the spectrum of the latter is filled with absorption lines would induce us to believe—quite apart from any question as to the temperature of the surface—that the amount of light leaving the star is less per unit surface for stars giving a solar spectrum, than for stars which only show hydrogen lines. The result of calculation showed that on the average the density of the solar stars was fifteen times greater than the density of the hydrogen stars;[1] but this number was founded on information which, as regards the nature of the spectrum emitted, was, in many cases, deficient. Pending a renewed inquiry into this important subject, I am struck by the slight systematic difference shown between the stars of different spectroscopic types. A density fifteen times as great means, for the same mass, a diameter reduced in the value of 2½ to 1, and even this difference would

[1] PROCTOR-RANYARD, *Old and New Astronomy*.

disappear if the hydrogen stars had an emissive power six times as great as that of the Sun.

The differences of density shown by individual stars of the same spectral type are considerably greater. Thus γ *Leonis* gives a spectrum almost identical with that of *Arcturus*, which is generally considered to belong to a later period than the solar stage; its density on the assumption of equal emissive powers is 0.0002 as compared with that of the Sun. It is not possible to admit an emissive power 300 times as great as that of the Sun, and hence the density of γ *Leonis* must be very considerably less than that of the Sun. As far as the observations go, the stars which are purely hydrogen stars show smaller variations in density than the solar stars, and have a density which is unmistakably smaller, but great variations are found in the intermediate stages in which the calcium lines are prominent. η *Cassiopeiae*, for instance, has a density almost equal to that of the Sun, being the third in order of density of all known stars, while γ *Virginis*, giving a similar spectrum, has a density sixty times smaller. The difficulty which, in the case of the binaries we have just discussed, arises from our ignorance of the intrinsic brightness of their surface, is overcome in the case of another set of close double stars, the so-called *Algol* variables. These binaries are characterized by the fact that they consist of two stars of unequal brightness, one of which passes periodically in front of the other so as to produce a variation in the combined brightness of the stars. But I must resist the temptation of entering into a detailed description of the peculiarities of these interesting stars, and content myself by referring to the conclusions of A. W. Roberts and H. N. Russell, which have given an average density considerably less than that of the Sun. Thus Roberts finds for the mean of four of these variables 0.18 for the greatest density consistent with the observations, while the corresponding density found by Russell for the average of seventeen variables is 0.20, closely agreeing with the former result. This means a density equal to about one-eighth of the solar density. For the fainter component of *S Velorum*, the greatest possible density is only 0.03 as compared with the Sun. The spectra of these stars all seem to

belong to the pre-solar type, and their low density therefore confirms the results arrived at from the consideration of other binaries, though we should not lose sight of the fact that the average density of the stars of the solar type seems to be considerably less than that of the Sun itself, so that the average density of the *Algol* variables is not much less than half the average density of the solar stars.

Interesting facts are brought to light when we investigate the distribution of types of spectra in different parts of the heavens. Such investigations are subject to the dangers accompanying all statistical inquiries, especially when the discussion must, to a great extent, turn on the differentiation between accidental and systematic deviations from the average. We owe a spectroscopic survey of the sky to Professor E. C. Pickering, who, with instruments belonging to the Henry Draper Memorial equipment, classified the spectra of 10,345 stars north of 25° of southern declination, and has since completed the investigation for stars which lie farther south. He has very fully discussed the results of the first of these surveys. Almost exactly half belong to the hydrogen type, 10 per cent. to the intermediate or calcium type, 12 per cent. to the solar type, and 25 per cent. to the *Arcturus* type. If we only take the stars which are brighter than magnitude 6.25, the hydrogen type includes relatively more, viz., 61 per cent., while only 5 per cent. belong to the solar type. The percentage of the Arcturian type is reduced to 18. If the region considered is divided into two equal portions, one lying as much as possible along the Milky Way, and the other away from it, it is found that, of all the stars considered, the Milky Way shows a preference for the stars of the hydrogen type, 3,560 of these stars being mapped in the region of the Milky Way, and 1,658 away from it; the corresponding numbers for the calcium type are 650 and 430. Neither the solar stars nor those having a spectrum similar to that of *Arcturus* show, when their total number is considered, a preference for any particular part of the sky. Out of the total number of stars, 6,252 belonged to the portion of the sky which included, and 4,095 to that portion which did not include, the Milky Way.

When we consider the distribution of stars of different magnitudes, we find that the hydrogen stars, which are of the fourth magnitude and brighter, seem to be distributed pretty evenly all over the heavens, and that it is only the weaker stars which show this effect of clustering in the regions of the Milky Way. The brighter solar stars, on the other hand, seem to be relatively more frequent in the Milky Way than away from it, and the more even distribution shown by the solar and Arcturian stars seems rather due to the fact that the stars of smaller magnitudes belonging to these types are chiefly found in the regions which lie away from the Milky Way. Thus, taking the stars down to the sixth magnitude, we find the numbers in the district of the Milky Way, and away from it, for the hydrogen stars to be 152 and 84, while for the Arcturian stars it is 453 and 341, in both cases an increased number in the Milky Way.

The results obtained by Dr. Frank McClean, who confined himself to stars above the 3.5 magnitude in both hemispheres, are not altogether in accordance with the above. Out of a total number of 276 stars, 30 per cent. only were found to belong to the hydrogen type, 17 per cent. to the *Procyon* or calcium type, while 31 per cent. were Arcturian or solar. An unusually large number, viz., 32 per cent., were classed as helium stars, and this leads to the supposition that a number of stars which figure in Pickering's list as hydrogen stars belong really to the helium subdivision. These helium stars show a very decided tendency to cluster in the Milky Way, but the distribution of the other types is, according to McClean, remarkably uniform. Thus, dividing the sky into two equal portions, the first of which includes the Milky Way, the numbers for the hydrogen type are 20 and 16, a very slight excess in favor of the Milky Way. For the calcium type the numbers are 21 and 27, and for the solar and Arcturian types combined, 45 and 40. The total number of stars of the hydrogen and calcium types is thus remarkably nearly equal to that of the solar and Arcturian types.

A promising line of inquiry has been entered upon by Mr. W. H. S. Monck, who finds that the apparent proper motion of solar and Arcturian stars is considerably greater than that of the Sirian

stars. Out of over 5,000 of the latter stars found in the *Draper Catalogue*, 225 are known to have a proper motion of not less than one tenth of a second per annum in one or other of the elements; this gives a percentage of less than 4.5. On the other hand, the percentage of solar and Arcturian stars having the same proper motion is 20 and 15 respectively. This points to the fact that the hydrogen stars are farther away than the solar and Arcturian stars of equal magnitude; or that, on the supposition of an equal distribution in space, and equal average real motion, the hydrogen stars are more luminous, a fact which is quite in agreement with our previous conclusions.

But a novel and unexpected result is the position of the Capellan or solar stars as being the nearest to us, while the Arcturian stars are intermediate between them and the hydrogen stars. Mr. Monck concludes that the Arcturian stars are not cooled-down Capellans, but we shall find that if the explanation I have suggested as to the higher temperature and greater luminosity of the hydrogen stars is correct, the apparently anomalous position of the Arcturian stars is readily explained.

I have intentionally confined myself to a detailed discussion of only two of Secchi's types of stars, but must now briefly refer to other celestial bodies, so that we may be able to obtain a general view of the evidence on which the theory of stellar evolution rests.

There are two kinds of nebulous bodies which may be distinguished by their spectra. One of them, of which the nebula in *Orion* may be taken as a specimen, shows us bright lines of hydrogen, of helium, and of some unknown substance. The second kind, of which the nebula in *Andromeda* is a conspicuous example, apparently give a continuous spectrum, which is weakened so much by spectroscopic dispersion that it is extremely difficult to form a definite judgment as to the nature of the light emitted. While Professor Scheiner believes that he has obtained by photography the absorption lines corresponding to the darkest groups of solar lines, the observations of Sir William and Lady Huggins seem to give very strong evidence of the presence of bright lines in the spectra of these bodies. It is to be hoped that

the matter may soon be cleared up, as these nebulæ include all those of spiral form, in which local condensations occur, suggesting a similarity to what we may suppose to be the origin of the formation of more compact celestial bodies. The gaseous nebulæ of the *Orion* kind are nearly all situated close to the Milky Way, while the nebulæ having the *Andromeda* character show the opposite behavior, and obviously avoid the plane of the Galaxy. Passing on to bodies which appear to be intermediate in character between stars and nebulæ, we also find them confined to the Milky Way. These bodies, as seen through a telescope, appear to be stars, but their light, when resolved by the spectroscope, shows bright lines, either alone or in conjunction with dark lines. Their distribution along the Milky Way is irregular, and they tend to cluster together in certain parts of it. These bright line stars vary to some extent in composition; they show as a rule the lines of hydrogen and some unknown lines. Helium appears in some, but not in all of them. There appears to be a gradual transition from these bright line stars to the helium stars, which also are chiefly found in the neighborhood of the Milky Way, and many of which are grouped together; thus all the bright stars of *Orion*, except *Betelgeuze*, and most of the weaker stars are helium stars. The high temperature of these helium stars is testified by the presence of oxygen lines, first identified by Dr. F. McClean, the particular spectrum of oxygen which appears in them being only obtainable in terrestrial oxygen by very intense sparks. The oxygen lines which, for instance, are found in the solar spectrum undoubtedly belong to a more complex molecule, and a lower temperature. There is again, apparently, a continuous transition from the helium stars to the hydrogen, calcium, solar, and Arcturian stars which I have described. The remaining types of spectra belong to lower temperatures still, as in place of the metallic lines, or in addition to them, certain bands appear, which experiments show us invariably belong to lower temperatures than the lines of the same element. Secchi's Type III possesses bands, as to the identity of which there is no consensus of opinion. These stars also cluster in the Milky Way, and a majority of them have the peculiarity of being variable in

intensity. The variations are of longer period than that of the *Algol* variables, and at the maximum the bright lines of hydrogen are seen in many cases. Lockyer holds that the carbon bands appear as bright lines, and this view, as far as I am able to judge of the evidence, is probably correct.

Nothing is known as to the reason of the light-variation, but among the causes which can produce the same effect at regularly recurring intervals there is only one which we can at present apply to celestial bodies with any show of reason, and that is the orbital revolution of the bodies around each other; but this question lies beyond the range of our present discussion.

The bands shown by Secchi's fourth type of spectra have been identified, and belong to carbon. The stars showing these spectra are all weak, but over 200 are known. Dunér, to whom we owe the first systematic investigation of these stars, has shown that they also congregate in the Milky Way, not only absolutely, but also relatively to the other stars. Mr. T. E. Espin has further investigated and confirmed this point. Out of a total of 224, the region within ten degrees of the Milky Way includes 123, or more than half, while 74 per cent. lie within twenty degrees of it.

We can only form vague guesses as to the manner in which matter was originally spread through space and has gradually condensed, probably, though not necessarily, through an intermediate nebular stage into the numerous luminous spherical bodies which we observe at night. If an evolutionary process has been going on, which is similar for all stars, there is little doubt that from the bright line stars down to the solar stars, the order has been: (1) helium or *Orion* stars, (2) hydrogen or Sirian stars, (3) calcium or *Procyon* stars, (4) solar or Capellan stars. The Arcturian stars are placed by most observers after the solar stars, but as mentioned above, the researches of Monck seem to bring them to an earlier stage of evolution. Opinions are divided as to the proper place to assign to the stars of the third type. There is, on the one hand, no definite boundary line between them and the Arcturian stars, the transition being gradual; the evidence of an evolution from the second or solar

to the third type is as strong as that which is generally recognized to indicate an evolutionary process from the Sirian to the solar type. On the other hand, the facts that these stars are nearly all variable, that they probably contain bright lines, and that they aggregate in the Milky Way, lend force to Lockyer's contention that the stars are in an early state of formation, and that the low temperature of their absorbing layer indicates that it is still rising in temperature. The carbon stars of the fourth type have been uniformly placed at the end of the succession of spectroscopic changes, though, according to many astronomers, this last stage does not succeed the third type stage, but follows the solar stage. These authorities would either, like Lockyer, remove the third type into the early features of a star's history, or derive it independently from the solar stage. According to this latter view, stars having reached the solar stage would bifurcate, and, according to their chemical composition, develop either the spectrum of the third or that of the fourth type.

The views I have expressed, and suggestions I have made in the previous pages, have led me to the following succession of events, as being in harmony with observed facts.

We may start from matter distributed with approximate uniformity through space, and leave out of account the question whether that original matter was in the form of our present elements, or in some primordial state, out of which our elements have been formed. If the latter case, the conditions must have been such that in different portions of space this primordial matter would condense into our elements, nearly, though probably not absolutely, in the same relative quantities. The formation of the elements I assume to take place simultaneously with the condensation into larger conglomerations.

The first point I want to draw attention to is that the effect of the first condensation must have been accompanied by a rejection of helium, hydrogen, and other light gases, because the development of heat which accompanies the early agglomeration, and raises the temperature of the gaseous bodies, must through the known laws of expansion increase their volume. The gravitation toward the condensed portions of matter not

being sufficient to retain the hydrogen, it will diffuse and tend to spread through the adjoining portions of space. In regions where there is no violent motion, it appears to me that matter will tend to concentrate itself around certain nuclei, which will begin to attract each other and clash together. A number of stars will then form, and these stars will at first not contain hydrogen or helium in appreciable quantities. These lighter gases will be left behind as nebulous masses, because it has been shown that unless the gravitation toward the center of a celestial body exceeds a certain value, light gases cannot form a permanent constituent of the atmosphere. This seems to be a not unlikely explanation of the gaseous nebulæ. These bodies are by observation found to be connected with stellar clusters, as has been shown by the remarkable photographs of the nebulous regions surrounding the *Pleiades*, and the spectroscopic investigation of the great nebula of *Orion*.

As soon as a star has grown sufficiently to be capable of retaining the hydrogen and helium, atmospheres of these gases will form around the stars, and the temperature of the photosphere will rise to its maximum. A process of diffusion of hydrogen and helium into the star will at once begin, and may be helped by the process of absorption which has already been alluded to. The helium will be retained first, as it is denser than hydrogen, and we may therefore expect to find helium stars showing the hydrogen lines either weakly or not at all. As a star grows in size the hydrogen will be more and more condensed on its surface from the outside, and we may get a considerable atmosphere of that gas forming around the star. The helium which has first condensed will also first diffuse toward the inside, and we then get the typical hydrogen star. The process of diffusion of the hydrogen, helped quite probably by an absorption due to molecular action, will continue to go on until a stage of equilibrium is reached, in which there is some, but possibly very little, hydrogen near the surface of the star.

It may be objected that the above explanation leaves out of account the peculiar behavior of calcium in the intermediate stage between the hydrogen and solar star. My answer is that

the same objection applies to all other explanations. We must at present accept it as a fact that there is a peculiar connection between some of the lines of calcium and the hydrogen line. This is shown by the phenomena which take place on the Sun, as strongly as by those which take place in the stars. The connection may be a chemical one, or may have a hitherto unsuspected origin. I have spoken of these lines which Fraunhofer designated by H and K as "calcium" lines, as so far we have only been able to obtain these lines when there was ground for the supposition that calcium was present. At the same time I do not think many spectroscopists would be surprised if it were found that these lines did not belong to the metal calcium at all. That was the opinion arrived at by Stas for reasons which to my mind are insufficient, but the opinion may ultimately turn out to be correct, even though for the moment experiment does not lend much countenance to it.[1] At present, therefore, we must content ourselves with accepting the peculiar behavior of these lines and their connection with hydrogen as an observed fact, but the acknowledged want of a sufficient explanation of the fact cannot be quoted as an objection to any particular view on the connection between different types of spectra.

A word may also be said as to the peculiarity of helium, which does not show its presence among the Fraunhofer lines, although it is known to be present in the Sun, because its lines appear bright in the chromosphere. There are two causes which may prevent a line from being visible as an absorption line: the vibration which gives rise to the line may be too nearly homogeneous, or it may be too intense.

It has never to my knowledge been pointed out, though it is obviously true, that an absolutely homogeneous vibration can never give rise to an absorption line in instruments of finite resolving powers. That is a possible, though perhaps not a very probable, explanation of our failure to detect helium among the

[1] (Note added February 1903.) The fact that the difference in the wave-numbers of H and K is nearly double that between the first and second line of the characteristic calcium triplets weighs strongly in favor of the lines being due to calcium. While writing the above I had for the moment forgotten this argument.

absorption lines. The second explanation is contrary to what is generally accepted as true, but only because writers are accustomed to consider the radiation from the solar photosphere to be the radiation of a perfectly black body. If the drops or liquid masses which form the photosphere have any power to reflect or scatter light, such as for instance we know rain drops to possess, the solar radiation need not, as regards intensity, exceed, say, half that due to a black body of the same temperature. It would then become quite possible for a *cooler* body in front either not to show any absorption lines, or even to show them as *bright* lines.[1] The absence among the Fraunhofer lines of the high temperature radiations observed in the chromosphere is probably due to this cause.

Returning now to the secular changes in the stars, I may point out the distinctive features of the views which I have suggested. These views, in the first instance, open out the possibility of much greater variations in their life-history than has generally been admitted. The amount of hydrogen, according to my present view, which a star is able to condense depends on its mass, and on the amount of hydrogen which happens to be present in the neighborhood. Whatever there was originally may already have been drawn toward a previously formed and bigger star. Hence the possibility that a star may form and never pass through the hydrogen stage. Even when the star has as small a density as γ *Leonis* probably has, it may give the spectrum of a solar star, simply from the want of a hydrogen atmosphere. On the other hand, there may be stars which, having attracted a large quantity of hydrogen, but being of comparatively small total mass and small size, are not able to absorb the gas completely, and may remain hydrogen stars without passing through the solar stage at all. Finally, the difference between the Arcturian and solar star may not be one of age at all, but of mass. If the Arcturian star is one which is bigger, it will be able to absorb the hydrogen more completely, and the final

[1](Note added February 1903.) I have obtained interesting results by a mathematical discussion of the phenomena of radiation through a foggy atmosphere. These I hope soon to be able to publish in this JOURNAL.

state of equilibrium will be such that the hydrogen lines will be thinner than in the Capellan or solar star.

This seems a plausible explanation of the results of Mr. Monck, who finds the Arcturian star to have a smaller proper motion than the solar stars. They are, if my views are true, brighter though cooler, because their surface is larger, and hence, on the average, they are farther away. The theory, if I may call it so, also gives an explanation of a very curious fact, which I venture to think has not so far been satisfactorily accounted for. In the case of double stars, it is often found that the brighter one is yellow and gives a solar spectrum, while the smaller one is blue and gives a hydrogen spectrum. The larger one, though it may originally have attracted more hydrogen to itself, will be able to absorb it more rapidly, and thus pass through the stages of spectroscopic evolution more quickly.

There is a marked difference between this explanation and that advocated by Sir William Huggins, according to whom a star of small mass would run rapidly through the various stages of evolution. We both agree, of course, in the main fact, that a small mass would lose heat more rapidly, but, according to the views here put forward, there is a counterbalancing tendency in the fact that a large mass would absorb the hydrogen more quickly, and therefore show a more rapid tendency to pass from the state in which it gives a hydrogen spectrum to the state in which the metallic lines become prominent.

I will not discuss the question of the spectra of the third and fourth types, for the reason that we have not sufficient data to form any decided opinion about them. As regards the spectra of the third type, it has already been mentioned that opinions differ as to whether their position is anterior to the hydrogen or posterior to the solar star, and there are valid arguments on both sides. The carbon stars are really disconnected from the others, and though they may ultimately be found to have a place in a general system of classification, there is at present nothing to show that they are not *sui generis* and the result of condensation in a space where carbonaceous matter happened to

be abundant. It has already been mentioned that they seem to show bright lines, and therefore probably belong to an early, rather than a late, stage of condensation. I know that the great principle of uniformity will be quoted against any supposition that a particular class of stars is essentially different in its composition from others, but I believe, on the other hand, that the skies bear ample evidence of real differences in composition. There is only need to mention, for instance, the dark bodies which by their passage in front of companion stars, produce the variation of light of the *Algol* variables. The obscuring stars have a density considerably less than that of water, and, as their temperature is low, they must be composed of elements differing widely from those which make up the Earth or the Sun. I cannot therefore admit the validity of an argument based on the so-called law of uniformity, which has always proved a fallacious guide.

Examples are plentiful in the history of science where the law of uniformity might have been quoted, and has been quoted, in support of obsolete moribund theories. Thus the savage, knowing that fire can be made by intelligent hands, unconsciously applied the law of uniformity to conclude that the lightning which set fire to the forests was caused by intelligent beings, surpassing him in grandeur as much as a lightning flash surpassed the feeble fire he could strike himself. When he saw the Sun apparently moving in his orbit, he was compelled by the law of uniformity to conclude that an intelligent being must carry that body in a chariot. When the mediæval magician felt that everything in this world seemed created and centered around man; when, moreover, he saw the Moon obviously describe an orbit around the Earth, he could quote the law of uniformity against the Copernican doctrine, which needlessly removed the center of the universe from what to him seemed its evident position. Even in more recent times false analogies, and conscious or unconscious appeals to the law of uniformity, have been constant sources of deception.

We are led by pure reasoning, and without any consideration of imaginary laws, to consider the universe to be in the state of

a clockwork which is running down. We can form some idea guided by our experience and observation, how a star may have formed and may pass through its various stages to extinction; but to say that all stars must necessarily pass through the same stages, to conclude that *Sirius* will ever look like *Arcturus*, is to put ourselves in the position of one, who having discovered that there is a certain law which apparently connects the ages of children with their height, calls the law of uniformity to witness that a man who is five feet high is necessarily younger than one who measures six. If the law of uniformity had reigned at creation, there could have been no life, for there can only be uniformity in death; but if there were sufficient diversity of position, of mixture, and of composition, to allow of aggregations of matter culminating in the formation of worlds, we may be sure that we shall be able to trace that diversity in the present composition of the stellar system. The universe shows law, order, and regularity, but it refuses to be forced from birth to death through a single channel. There is uniformity no doubt, but it is a uniformity which at all times, and in all places, is relieved by endless variety.

A NEW VARIABLE STAR OF UNUSUALLY SHORT PERIOD.[1]

By G. MÜLLER and P. KEMPF.

IN the course of the zone observations for Part III of the Potsdam *Photometric Durchmusterung* it appeared that the two regular measures of the brightness of the star of the seventh magnitude *B.D.* $+56°$ 1400 ($\alpha = 9^h\ 36^m\ 44^s$; $\delta = 56°\ 24'.6$ [1900]) in 1899 and 1901 differed from each other by an amount greater than that considered permissible for this *Durchmusterung*. Although the revision observations in the period from April 19 to June 4, 1902, left no doubt as to the variability of the star, they nevertheless gave no indication as to the character of the variation. The measures were continued until the end of July, 1902, and later resumed after the appearance of the star in the eastern heaven, without our succeeding in detecting the character of the variation. It was not until the 13th of January of this year, when the star was several times observed during a period of three hours in the course of the evening, that a decline and rise of the light could be established and the time of minimum approximately derived as about $9^h\ 20^m$ Potsdam M. T. This showed that the light changes occurred in a comparatively short time, and the star was therefore observed on the same night at intervals of ten minutes until shortly before sunrise. A definitive conclusion as to the still somewhat doubtful character of the light-variation was reached through the observations of January 14, which were carried on without interruption from $4^h\ 48^m$ until $9^h\ 19^m$ Potsdam M. T. They furnished a complete view of the whole light-curve, and thus led to the discovery of a variable star with the extraordinarily short period of only four hours, the shortest so far known.

All of our measures of the new variable are summarized in tabular form below. The first six values are taken from the zone

[1] Translated from advance proofs, furnished by the authors, of a paper to appear in the *Sitzungsberichte der K. Akad. der Wiss. zu Berlin*.

observations for Part III of the *Potsdam Durchmusterung* in which the variable is compared with fundamental stars. Fundamental stars were also used for comparison at the next three observations on June 10 and 25, 1902. But from June 28, 1902, the near-by star *B. D.* + 54°.1329 ($\alpha = 9^h\ 41^m\ 44^s$; $\delta = 54°\ 43'.7$ [1900]) served exclusively as the comparison star. For its magnitude we obtain from comparisons with fundamental stars the following ten values: 7.65, 7.73, 7.81, 7.77, 7.68, 7.82, 7.64, 7.63, 7.75, and 7.85, the mean of which is 7.73.

The first five columns of the following table contain successively the date of observation, the local sidereal time, the Greenwich Mean Time, the designation of the observer, and the magnitude of the variable as derived from the measures. The last three columns of the table will be explained later.

Date		Sid. Time	G.M.T.	Obs.	Mag.	C.	O.-C.	Epoch
1899, May	29	$15^h\ 16^m$	$9^h\ 55^m$	K	7.76	7.94	−18	−7942
1901, January	17	4 50	8 12	M	8.33	8.04	+29	−4358
1902, April	19	14 13	11 32	M	8.12	8.18	− 6	−1617
April	22	14 2	11 9	K	8.58	8.40	+18	−1599
June	2	15 29	9 55	M	7.89	7.90	− 1	−1354
June	4	15 22	9 40	K	7.97	7.90	+ 7	−1342
June	10	15 15	9 10	M	7.84	8.01	−17	−1306
June	25	15 47	8 43	M	8.18	8.29	−11	−1216
		16 3	8 59	K	8.19	8.17	+ 2	
June	28	16 39	9 23	M	7.87	8.05	−18	−1198
June	29	16 42	9 22	K	8.06	8.07	− 1	−1192
July	5	17 17	9 33	M	8.13	8.05	+ 8	−1156
		17 21	9 37	K	8.11	8.03	+ 8	
July	6	16 53	9 5	M	8.28	8.23	+ 5	−1150
		16 58	9 10	M	8.18	8.20	− 2	
July	12	16 38	8 27	K	8.36	8.55	−19	−1115
		16 46	8 35	M	8.29	8.57	−28	−1114
July	15	17 19	8 56	M	8.43	8.40	+ 3	−1096
		17 24	9 1	M	8.27	8.35	− 8	
July	16	17 7	8 40	M	8.44	8.57	−13	−1090
		17 17	8 50	M	8.51	8.48	+ 3	
		17 23	8 56	M	8.60	8.41	+19	
July	19	18 6	9 27	M	8.30	8.19	+11	−1072
		18 11	9 32	M	8.29	8.16	+13	
July	21	16 57	8 10	M	8.14	8.12	+ 2	−1061
		17 2	8 15	M	8.04	8.16	−12	
		17 7	8 20	M	8.21	8.22	− 1	
July	28	17 10	7 56	M	8.07	8.00	+ 7	−1019
		17 15	8 1	M	7.91	8.02	−11	
		17 30	8 16	M	8.09	8.11	− 2	
November	27	3 3	9 47	M	8.11	7.90	+21	− 287

Date	Sid. Time	G.M.T.	Obs.	Mag.	C.	O.-C.	Epoch
1902, December 11	$3^h 11^m$	9^h 0	K	8.18	8.04	+14	− 203
December 12	3 7	8 52	M	8.17	8.08	+ 9	− 197
	3 13	8 58	K	8.09	8.05	+ 4	
December 13	3 36	9 17	M	8.02	7.98	+ 4	− 191
	3 43	9 24	K	8.03	7.95	+ 8	
December 14	3 53	9 30	M	7.81	7.94	−13	− 185
	3 59	9 36	K	7.91	7.92	− 1	
1903, January 12	4 23	8 6	M	8.16	8.26	−10	− 12
	4 31	8 14	K	8.34	8.41	− 7	
January 13	3 26	7 5	M	7.90	7.93	− 3	− 6
	3 29	7 8	K	7.90	7.93	− 3	
	4 20	7 59	K	8.06	8.15	− 9	
	4 25	8 4	M	8.15	8.21	− 6	
	5 18	8 57	K	8.35	8.32	+ 3	− 5
	5 24	9 3	M	8.19	8.27	− 8	
	6 21	10 0	M	8.00	7.96	+ 4	
	6 28	10 7	K	7.99	7.94	+ 5	
	13 0	16 38	M	8.53	8.52	+ 1	− 3
	13 3	16 41	M	8.46	8.49	− 3	
	13 17	16 55	M	(8.63)	8.34	+29	
	13 28	17 6	M	8.34	8.25	+ 9	
	13 35	17 13	M	8.28	8.20	+ 8	
	13 47	17 25	M	8.04	8.13	− 9	
	13 59	17 37	M	8.05	8.06	− 1	
	14 12	17 50	M	8.03	8.00	+ 3	
	14 21	17 59	M	7.99	7.97	+ 2	
	14 33	18 11	M	7.86	7.93	− 7	
	14 44	18 22	M	7.93	7.91	+ 2	
January 14	0 20	3 56	M	8.27	8.12	+15	− 1
	0 30	4 6	K	8.20	8.22	− 2	
	0 35	4 11	K	8.35	8.29	+ 6	
	0 43	4 19	M	8.38	8.45	− 7	
	0 49	4 25	M	8.64	8.55	+ 9	
	0 55	4 31	K	8.57	8.58	− 1	
	1 4	4 40	K	8.55	8.51	+ 4	0
	1 8	4 44	M	8.55	8.47	+ 8	
	1 15	4 51	M	8.38	8.39	− 1	
	1 22	4 58	K	8.30	8.32	− 2	
	1 26	5 2	K	8.33	8.29	+ 4	
	1 35	5 11	M	8.16	8.23	− 7	
	1 40	5 16	M	8.31	8.19	+12	
	1 45	5 21	K	8.22	8.16	+ 6	
	1 51	5 27	K	8.09	8.12	− 3	
	1 57	5 33	M	8.00	8.08	− 8	
	2 3	5 39	M	8.09	8.05	+ 4	
	2 10	5 46	K	8.02	8.02	0	
	2 15	5 51	K	8.15	8.00	+15	
	2 22	5 58	M	7.99	7.98	+ 1	
	2 29	6 5	M	7.98	7.95	+ 3	
	2 36	6 12	K	7.90	7.93	− 3	
	2 44	6 20	K	7.89	7.91	− 2	
	2 51	6 27	M	7.83	7.90	− 7	
	2 59	6 35	M	7.82	7.90	− 8	
	3 6	6 42	K	7.91	7.90	+ 1	
	3 12	6 48	K	7.88	7.90	− 2	

Date		Sid. Time	G.M.T.	Obs.	Mag.	C.	O.-C.	Epoch
1903, January	14	$3^h\ 22^m$	$6^h\ 57^m$	M	7.95	7.91	+ 4	
		3 28	7 3	M	8.00	7.92	+ 8	
		3 34	7 9	K	7.94	7.93	+ 1	
		3 43	7 18	K	8.06	7.95	+11	
		3 49	7 24	M	8.04	7.97	+ 7	
		3 55	7 30	M	8.03	7.99	+ 4	
		4 1	7 36	K	8.05	8.02	+ 3	0
		4 7	7 42	K	7.96	8.04	− 8	
		4 14	7 49	M	8.10	8.08	+ 2	
		4 19	7 54	M	8.21	8.11	+10	
		4 23	7 58	K	8.10	8.14	− 4	
		4 38	8 13	M	8.42	8.33	+ 9	
		4 42	8 17	M	8.43	8.41	+ 2	
		4 44	8 19	K	8.45	8.45	0	
		4 52	8 27	K	8.59	8.56	+ 3	
January	17	0 31	3 55	K	8.02	8.09	− 7	17
		0 38	4 2	K	8.16	8.14	+ 2	
		0 44	4 8	M	8.19	8.19	0	
		0 50	4 14	M	8.40	8.28	+12	
		0 54	4 18	K	8.35	8.35	0	
		1 0	4 24	K	8.41	8.48	− 7	
		1 5	4 29	M	8.50	8.55	− 5	
		1 12	4 36	M	8.56	8.58	− 2	18
		1 15	4 39	K	8.66	8.56	+10	
		1 22	4 46	K	(8.74)	8.49	+25	
		1 25	4 49	M	8.37	8.45	− 8	
		1 31	4 55	M	8.37	8.39	− 2	
		1 35	4 59	K	8.36	8.35	+ 1	
		1 41	5 5	K	8.35	8.30	+ 5	
		1 46	5 10	M	8.24	8.26	− 2	
		1 52	5 16	M	8.24	8.22	+ 2	
		1 55	5 19	K	8.05	8.20	−15	
		2 2	5 26	K	8.12	8.15	− 3	
		2 5	5 29	M	8.10	8.13	− 3	
		2 11	5 35	M	7.90	8.10	−20	
		2 15	5 39	K	7.99	8.07	− 8	
		2 20	5 44	K	8.09	8.05	+ 4	
		2 25	5 49	M	8.00	8.03	− 3	
		2 31	5 55	M	8.09	8.00	+ 9	
		2 36	6 0	K	7.97	7.98	− 1	
		2 42	6 6	K	7.94	7.96	− 2	
		2 48	6 12	M	7.91	7.94	− 3	
		2 55	6 19	M	7.88	7.92	− 4	
		3 3	6 27	K	7.89	7.91	− 2	
		3 10	6 34	K	7.90	7.90	0	
		3 17	6 41	M	7.83	7.90	− 7	
		3 23	6 47	M	8.00	7.90	+10	
		3 31	6 55	K	7.87	7.90	− 3	
		3 39	7 3	K	7.96	7.91	+ 5	
		3 45	7 9	M	8.07	7.93	+14	
		3 50	7 14	M	7.95	7.94	+ 1	
		3 56	7 20	K	8.01	7.95	+ 6	
		4 2	7 26	K	7.93	7.96	− 3	
		4 14	7 38	M	8.15	8.01	+14	
		4 19	7 43	M	8.11	8.03	+ 8	

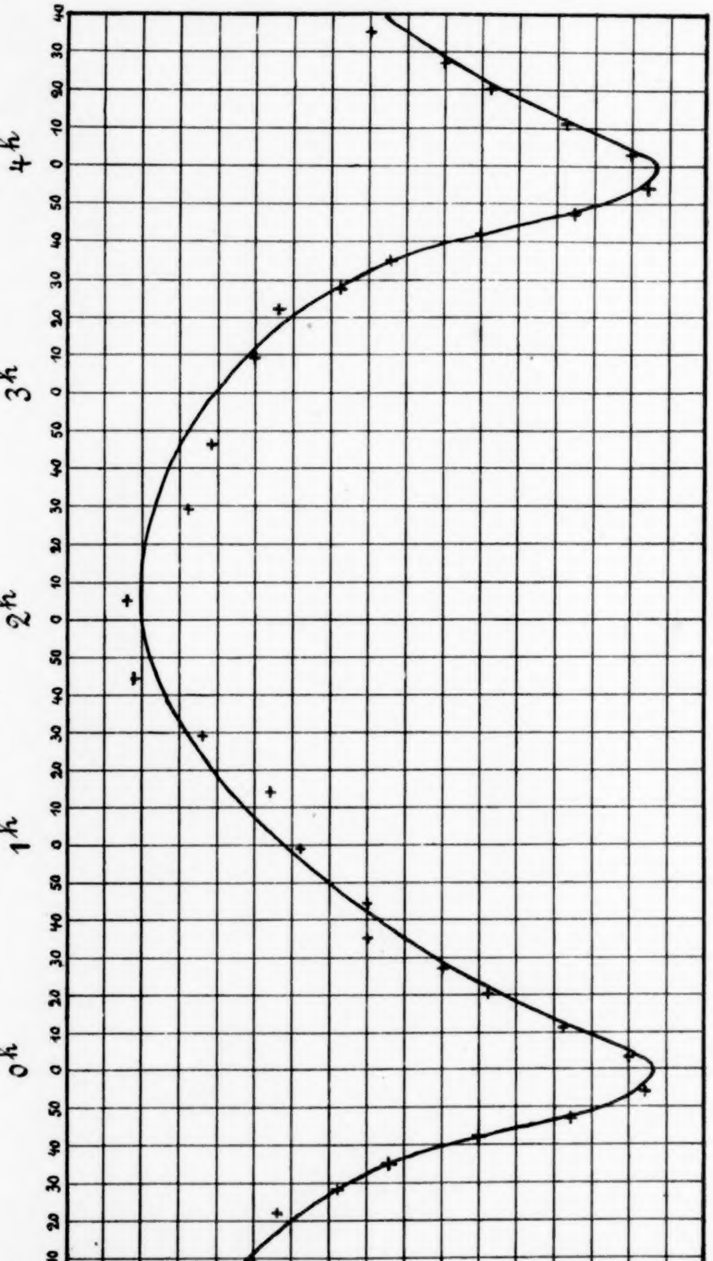
Date		Sid. Time	G.M.T.	Obs.	Mag.	C.	O.-C.	Epoch
1903, January	17	$4^h 26^m$	$7^h 50^m$	K	7.99	8.06	− 7	
		4 32	7 56	K	8.14	8.10	+ 4	
		4 37	8 1	M	8.08	8.13	− 5	
		4 43	8 7	M	8.28	8.18	+10	
		4 46	8 10	K	8.17	8.22	− 5	
		4 52	8 15	K	8.27	8.29	− 2	
		4 55	8 18	M	8.34	8.35	− 1	
		5 0	8 23	M	8.60	8.45	+15	
		5 3	8 26	K	8.58	8.52	+ 6	
		5 9	8 32	K	8.64	8.56	+ 8	
		5 13	8 36	M	8.61	8.58	+ 3	19
		5 18	8 41	M	8.37	8.54	−17	
		5 20	8 43	K	8.54	8.52	+ 2	
		5 26	8 49	K	8.34	8.45	−11	
		5 30	8 53	M	8.33	8.41	− 8	
		5 36	8 59	M	8.28	8.35	− 7	
		5 38	9 1	K	8.24	8.33	− 9	
		5 44	9 7	K	8.19	8.28	− 9	
		5 51	9 14	M	8.15	8.23	− 8	
		5 56	9 19	M	8.17	8.20	− 3	
January	18	4 30	7 50	M	8.01	8.06	− 5	24
		4 36	7 56	M	7.87	8.09	−22	
		4 39	7 59	K	7.99	8.11	−12	
		4 44	8 4	K	8.27	8.15	+12	
		4 48	8 8	M	8.07	8.18	−11	
		4 52	8 12	M	8.31	8.23	+ 8	
		4 55	8 15	K	8.24	8.28	− 4	
		5 1	8 20	K	8.36	8.37	− 1	
		5 4	8 23	M	8.45	8.43	+ 2	
		5 9	8 28	M	8.57	8.53	+ 4	
		5 12	8 31	K	8.45	8.56	−11	
		5 18	8 37	K	8.57	8.58	− 1	25
		5 21	8 40	M	(8.28)	8.56	−28	
		5 26	8 45	M	8.51	8.51	0	
		5 29	8 48	K	8.42	8.48	− 6	
		5 34	8 53	K	8.41	8.42	− 1	
		5 38	8 57	M	8.37	8.38	− 1	
		5 43	9 2	M	8.25	8.33	− 8	
		5 45	9 4	K	8.26	8.31	− 5	
		5 51	9 10	K	8.14	8.27	−13	

The graphical representation of the observations of January 14, 17, and 18 gave the following four minima, the uncertainty of which can be estimated at 10 minutes at most:

Jan. 14	- - - -	$4^h 34^m$ G.M.T.
17	- - -	4 40
17	- - - -	8 31
18	- - -	8 34

The combination of these data yielded the first provisional elements of the variable:

$$\text{Min.} = 1903 \text{ Jan. } 14^d\ 4^h\ 34^m \text{ G.M.T.} + 4^h\ 0^m.0\ E.$$



LIGHT-CURVE OF *B. D.* + 56°.1400.

The uncertainty of the period cannot in any case be assumed to be greater than 1 minute, whence the true value must lie between $3^h\ 59^m$ and $4^h\ 1^m$.

The observations of the year 1902 can be utilized for correcting the first approximation of the period. Since the brightness of the star at minimum is approximately 8.6, it is at once evident that the observations of April 22 and July 16 were made very nearly at the time of a minimum; the uncertainty could hardly amount to more than from 20 to 25 minutes. Combining these two dates together and with the minimum of January 14, it can easily be proved that only the four following periods can come into consideration, each of which can have a play of not more than $\pm$ 0.03 minutes:

$$4^h\ 0^m.65\ ;\ 4^h\ 0^m.21\ ;\ 3^h\ 59^m.77\ ;\ 3^h\ 59^m.33.$$

It may be further shown, with the employment of a light-curve provisionally derived by comparison with the remaining observations of 1902, that the first and the last two of the above four values are to be rejected within their whole range, because they yield inadmissibly large deviations between the computed and observed brightness. The second value only remains, and its application is limited to the range from $0^m.20$ to $0^m.22$. We therefore assume as the second approximation, after a slight displacement of the epoch, the elements:

$$\text{Min.} = 1903\ \text{Jan.}\ 14^d\ 4^h\ 32^m\ \text{G.M.T.} + 4^h\ 0^m.21\ E.$$

This formula was now used for constructing the light-curve of the variable from the measures of January 12–18, the epochs of minima being computed by it, and the differences of time of the separate data of observation as compared with the previous mimimum were formed. In all 143 measures were employed for the light-curve; three of these (distinguished in the table of observations by parentheses) were excluded, as they were clearly affected by somewhat large errors of observations, and would have influenced the result too strongly. The remaining 140 measures were arranged in order of their distance from a minimum and were finally combined in twenty mean values, each from seven measures. These normal values are contained in the following table:

NORMALS.

Distance from Minimum	Observed Magnitude	Curve	O.—C.
$0^h\ 3^m$	8.55	8.56	−1
0 11	8.46	8.48	−2
0 20	8.36	8.38	−2
0 27	8.30	8.31	−1
0 35	8.20	8.25	−5
0 44	8.20	8.19	+1
0 59	8.11	8.10	+1
1 14	8.07	8.02	+5
1 29	7.98	7.96	+2
1 44	7.89	7.92	−3
2 5	7.88	7.90	−2
2 29	7.96	7.92	+4
2 46	7.99	7.95	+4
3 9	8.05	8.04	+1
3 22	8.08	8.11	−3
3 28	8.16	8.15	+1
3 35	8.23	8.23	0
3 42	8.35	8.35	0
3 47	8.47	8.46	+1
3 54	8.57	8.56	+1

The accompanying light-curve was drawn with the aid of these values. The magnitudes read off from the curve for every five minutes are collected in the following table:

TABLE OF MAGNITUDES.

Distance from Minimum	Magnitude	Distance from Minimum	Magnitude	Distance from Minimum	Magnitude	Distance from Minimum	Magnitude
$0^h\ 0^m$	8.58	$1^h\ 0^m$	8.09	$2^h\ 0^m$	7.90	$3^h\ 0^m$	8.00
0 5	8.54	1 5	8.06	2 5	7.90	3 5	8.02
0 10	8.49	1 10	8.04	2 10	7.90	3 10	8.04
0 15	8.43	1 15	8.02	2 15	7.90	3 15	8.07
0 20	8.38	1 20	8.00	2 20	7.90	3 20	8.10
0 25	8.33	1 25	7.98	2 25	7.91	3 25	8.13
0 30	8.29	1 30	7.96	2 30	7.92	3 30	8.17
0 35	8.25	1 35	7.94	2 35	7.93	3 35	8.23
0 40	8.22	1 40	7.93	2 40	7.94	3 40	8.31
0 45	8.18	1 45	7.92	2 45	7.95	3 45	8.41
0 50	8.15	1 50	7.91	2 50	7.96	3 50	8.52
0 55	8.12	1 55	7.90	2 55	7.98	3 55	8.56
1 0	8.09	2 0	7.90	3 0	8.00	4 0	8.58

The magnitudes taken from this table are given, along with the observed magnitudes, in the above table of the normal values in the column entitled "Curve." The differences between observation and computation are given in the last column.

As may be seen from the table of magnitudes, and still better from the drawing, the light-variation proceeds very rapidly around the time of minimum, the curve at minimum almost forming an acute angle. The decline to the least brightness is somewhat steeper than the rise thereafter, the two branches not being entirely symmetrical. The maximum is far less sharply pronounced than the minimum, but the observations appear to exclude the possibility that the star remains at its greatest brightness without change for some time; wherefore it cannot be regarded as of the *Algol* type. It is somewhat striking that the normal values at about an hour before the maximum, and similarly some time afterward, lie in general below the curve. The impression is given of a short pause at these times in the increase or decrease of the light, and as if the curve ought to be drawn with two depressions. It cannot be proven without a much greater amount of observed data whether these irregularities are actually real, or are to be assigned to uncertainty or prepossession during the observations. We have paid no attention to them at present.

It should be remarked further that the observations up to this time give no indication of a different brightness at the even and the odd minima. Any irregularity in the intervals between every two successive minima is equally impossible of recognition.

The definitive table of magnitudes was further employed for more closely limiting the second approximation of the period. Here the first two observations of 1899 and 1901 could be employed, the one of which must lie at the time of maximum and the other not far from a minimum. Different trials showed that the most probable value of the period is included between $4^h\ 0^m.210$ and $4^h\ 0^m.220$, and in fact the sum of the squares of the residuals was least for the values $0^m.212$ and $0^m.214$. We adopted the mean from these values, and assume as the most probable elements of the new variable at the present time:

Min. = 1903, January $14^d\ 4^h\ 32^m$ G. M. T. + $4^h\ 0^m\ 12^s.8$ E.

The error of the period can hardly be more than $0^s.5$, and the correction to it cannot be expected for a number of months. The last columns in the table of measures show how all the

observations are represented by the above period. The magnitudes as taken from the light-curve are there given, together with the deviations between observation and computation. The last column contains further the number of the epoch of the minimum preceding the observations in question, reckoned from the initial epoch 1903, January 14. On the whole, the representation may be considered as satisfactory — among the 181 observations there is no deviation greater than $0^m.29$.

The most rapid oscillations in brightness among the variables hitherto known are exhibited by two stars in the star cluster *ω Centauri*, which is rich in variables; their periods are $7^h\ 11^m.4$ and $7^h\ 42^m.8$. *S Antliae* follows next with a period of $7^h\ 46^m.8$. Periods between 8^h and 9^h are found for several variables in that cluster. *U Pegasi* should be finally mentioned, the period of which is given as $5^h\ 32^m.2$ in Chandler's third catalogue, but which exhibits secondary minima according to Pickering's investigations (*Harvard Circular* No. 23), and has a period of $8^h\ 59^m.7$.

The discovery of the new variable raises the question of the cause of this exceedingly rapid light-variation. We might first think with Zöllner of a rotating body whose surface possesses a very unequal distribution of brightness as a consequence of an advanced stage of cooling. This view is opposed by the white color of the star, for we may assume a yellowish or reddish color for all stars which are strongly cooled off. Another natural supposition would be that of a figure widely departing from a sphere, perhaps a long ellipsoid or a body similar to one of Darwin's figures of equilibrium, which rotate about one of the lesser axes. This explanation, however, would meet with difficulties, because it can hardly be possible to represent the particular form of light-curve observed, especially the very rapid changes of brightness at minimum and the very slow changes around the time of maximum.

We may finally consider the hypothesis that the light-variation is produced by two celestial bodies almost equal in size and luminosity whose surfaces are at a slight distance from each other, and which at times almost centrally occult each other in their revolution. In this case the observed light-curve can be

almost exactly represented by computation. The fact that the difference of brightness between maximum and minimum is somewhat less than $\frac{3}{4}$ mag. would indicate that one body is a little smaller than the other, or that the occultation is not quite central. On this hypothesis we have only one difficulty, and the not inconsiderable one, as to whether such a system is mechanically possible and can remain stable for any length of time. But in spectroscopic binaries we have already come to know systems the existence of which formerly had to be considered as doubtful on similar grounds, and it would perhaps be possible by more exhaustive theoretical investigations to demonstrate also the permissibility of the assumption of still closer double stars.

POTSDAM,
February 1903.

THE SPECTROSCOPIC BINARY *o PERSEI*.[1]

By H. C. Vogel.

The Astrophysical Journal for April 1902 (**15**, 214) contains a communication by Mr. W. S. Adams as to certain spectroscopic binaries recently found at the Yerkes Observatory. The following five observations were given of the star *o Persei* ($\alpha=3^h\ 38^m$; $\delta=31°\ 58'$):

1902,	February 19	+134km
	February 21	−77
	March 4	+128
	April 2	−117
	April 3	− 4

A few plates of the spectrum of this star, taken by Dr. Eberhard with Spectrograph IV attached to the photographic refractor (32.5 cm) of the Potsdam Observatory, showed that *o Persei* could also be successfully observed with this telescope with the employment of the large dispersion given by Spectrograph IV. Thus far eighteen spectrograms have been obtained by Dr. Eberhard, with the assistance of Dr. Scholz, and I have undertaken their measurement and discussion.

The star is of the fourth magnitude, and has an ill-defined spectrum of Class I*b*, on earlier plates of which, taken with less dispersion, I was able to measure sixteen lines, chiefly due to hydrogen and helium.[2] Even with low dispersion the lines were very weak and obscure, particularly those of hydrogen. With the higher dispersion of Spectrograph IV there was visible, in the part of spectrum investigated (from λ 4315 to λ 4495), the hydrogen line $H\gamma$ as a faint brightening in the continuous spectrum. The helium line λ 4472, which was also visible and measurable, was very weak and broad; the helium line λ4388, which was measurable on most of the plates, was similar to

[1] Translated from advance proofs, furnished by the author, of a paper to appear in the *Sitzungsberichte der K. Akad. zu Berlin*.

[2] *Publ. des Astrophys. Obs.*, **12**, 33.

λ 4472. On some of the plates there were indications of the presence of a line somewhat less ref[illegible]angible than λ4388, and of the *Mg* line at λ4481. This *Mg* line was indeed distinctly visible on several plates when they were viewed with a magnifying glass, but it disappeared under the higher magnification of the microscope. On some plates the helium line λ4388 (and also 4472) appeared fringed with bright edges, similar to the *Mg* line λ4352 and the hydrogen lines in certain stellar spectra. The measurements could therefore be made only upon the very broad, diffuse $H\gamma$ line and upon the two weak and broad lines of the clèveite gas at λ4388 and 4472. On account of the faintness of thé $H\gamma$ line in the spectrum of *o Persei*, I could not here employ the method I had previously used with advantage of increasing the accuracy of the measurements of the broad and ill-defined $H\gamma$ line of most spectra of Class I by covering the line with a somewhat narrower pointer, which was then moved back and forth until the diffuse edges of the $H\gamma$ line were equally distant to the right and left of the pointer. For the same reason the double thread could never be used during the measurements.

Dr. Eberhard took the greatest pains to suit the exposure time to the atmospheric conditions, and gave especial care in the development of the plates. The exposures varied from 30 to 60 minutes, averaging ;40 minutes. The slit-width was always 0.02mm.

I should further state that I derived the displacement of the lines in the stellar spectrum from measures of the distance of the three above-mentioned lines from neighboring lines of the *Fe* comparison spectrum. At least six settings were made on each star line, usually with a power of 20; the measurements were also often repeated with the use of different magnifications—from 10 to 35. The precaution was also always taken of not making the settings of the micrometer thread on the star line one after another, but with interruptions, in order that the eye should not be too greatly fatigued by the difficult measments, and in order to be free from any distinct *Auffassung* due to peculiarities of the photographic film. The smallest irregularities in the silver deposit may have a great effect on the weak

lines of the spectrum, and it is often difficult to obtain a correct *Auffassung*. To illustrate this further, I would say that with a power of 10 one of the lines on a certain plate appeared to be quite oblique to the direction of the length of the spectrum, while with higher powers, which clearly brought out the grain of the plate, it turned out that the impression of an oblique position of the line was produced by a small cross-mark which had been formed within the stellar line by the running together of several silver grains.

The table on p. 215 gives the velocities resulting from the displacements of the line $H\gamma$ (*a*), λ 4388 (*b*), and λ 4472 (*c*).

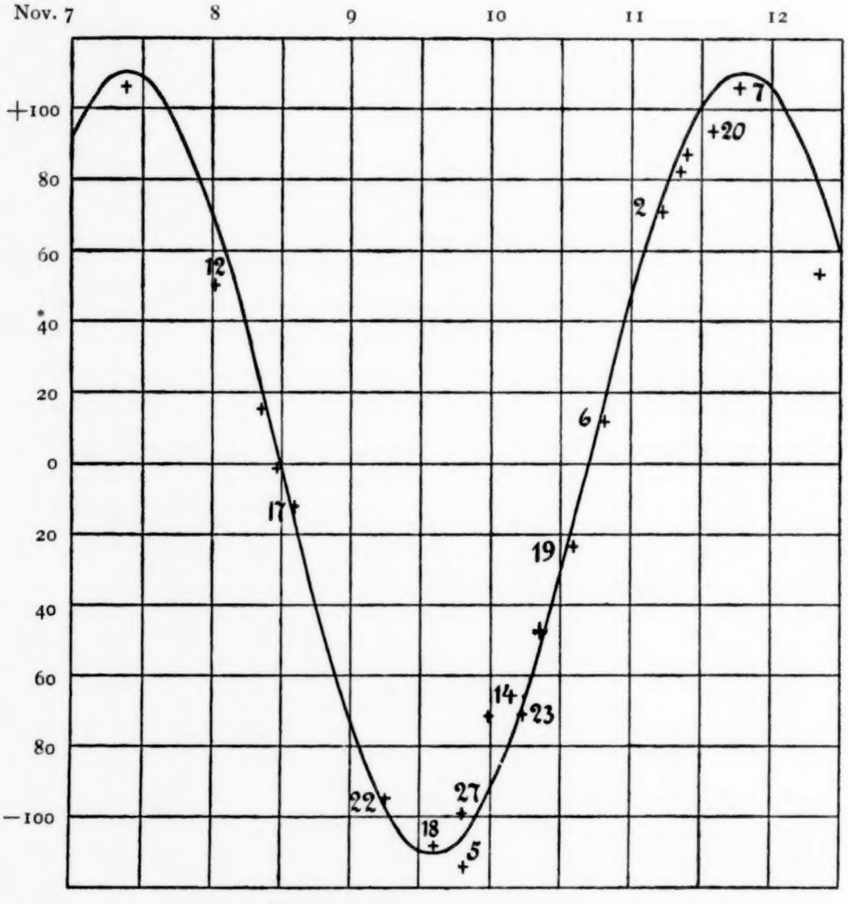
The arrangement of the table requires no further explanation. I have rounded the velocities off to kilometers, in conformity to the low degree of certainty of the measurements. A low relative weight was given to some of the measurements in the record of measures, which has been added in parentheses and was taken into account in forming the means. A : indicates less accuracy. In the last column 1 denotes very good, 4 bad.

On Plate No. 1269 a very fine double line is visible approximately at the position of the *Mg* line λ 4481; it would yield a velocity of -74 km. I regard this fine double line, however, as a defect on the plate, and have therefore not included that measure in the table. Something similar appears within the helium line λ 4472 on Plate 1271, where a fine line is visible somewhat less refrangible than the center of the line; but I must regard this also as a defect in the silver deposit. A velocity of -64 km would be derived from its measurement.

I now sought to combine the observations by a curve, and found that the period may be taken as 4.39 days. Its more accurate derivation will not be possible without further observations extending over a longer time. The assumption of 4.39 days has been sufficient, however, for the reduction of the different observations to the duration of the period, for the purpose of a more obvious graphical representation. It appeared further that there is no noticeable departure from a circular orbit, and the curve in the figure is therefore drawn, not so as to best include the observations, but to represent a circular orbit

Date (Central European Time)	Plate No.	Velocity Relative to Earth	Mean Values of the Velocity		Quality of Observation
			Rel. to Earth	Rel. to Sun	
1902, November $2^{d}.440$	1213	*a* + 51 km *c* + 71	+ 61 km	+ 71 km	2
5.431	1224	*a* −119 *b* −115 *c* −131	−122	−114	2
6.428	1228	*a* + 2 *b* + 12 *c* − 3	+ 4	+ 12	1–2
7.378	1234	*a* + 97 *b* + 87 *c* +113	+ 99	+106	1–2
8.374	1237	*a* + 14 *b* + 21 *c* − 9	+ 9	+ 16	2
8.462	1239	*a* 0 *b* + 17 *c* − 41	− 8	− 1	2
10.361	1242	*a* − 44 *c* − 65	− 55	− 49	2
11.354	1244	*a* + 85 *c* + 61(½)	+ 77	+ 82	3–4
11.401	1245	*a* + 85 *b* + 84 *c* + 77	+ 82	+ 87	1
12.413	1246	*a* + 47	+ 47	+ 52	3
14.355	1252	*a* − 70 *b* − 74 *c* − 80	− 75	− 71	2
17.363	1253	*a* − 28 *b* − 12 *c* − 1	− 14	− 12	1
18.379	1257	*a* −113 *b* −111 *c* −105	−110	−108	1
19.378	1262	*a* − 19 *b* − 24 *c* − 32(⅔)	− 24	− 23	1–2
20.364	1265	*a* + 77 *b* +104 *c* + 98	+ 93	+ 94	2
22.415	1269	*a* − 79 *b* −104: *c* −103	− 95	− 95	2–3
23.408	1271	*a* − 58 *b* − 79(½) *c* − 77	− 70	− 71	1–2
27.351	1282	*a* − 92 *b* −103 *c* − 93	− 96	− 99	1

computed with the above period and a maximum velocity of 110 km. The date of the transition from positive to a negative velocity is taken as November 8.50. There appears to be a



VELOCITY-CURVE OF *o Persei*.

The date is given for those observed points which were reduced to the portion of the curve from November 7 to November 12 with the period of 4.39 days.

slight difference indicated between the greatest positive and the greatest negative values, from which a negative value of several kilometers would follow for the motion of the system.

The accuracy of the observations, though indeed very small,

nevertheless turns out better than I had originally expected. On comparing the values for the single lines on a plate with the mean, the probable error of the velocity derived from one line on one plate comes out as ± 9 km; accordingly the probable error of the mean of the measures on one plate will be ± 5 km.

On a closer examination of the velocities derived from three lines on a plate the fact must nevertheless attract attention that, with the exception of those plates on which the lines exhibit a slight displacement, the velocities derived from the displacement of $H\gamma$ are almost always smaller, considered absolutely, than the mean value from the measures of the lines in the spectrum of the clèveite gas.

The derived velocities lie within 20 km on the four plates 1228, 1237, 1239, and 1253.

The velocities derived from the two helium lines are almost equal to that found from the $H\gamma$ line on the three plates 1234, 1245, 1257.

The $H\gamma$ line gives distinctly less values than the other lines on the nine plates, 1213, 1224, 1242, 1252, 1262, 1265, 1269, 1271, and 1282. The deviation averages 13 km.

The first exposure on November 11, No. 1244, may be excluded as too weak, and on Plate 1246 the motion is derived solely from $H\gamma$. The departure of the value from the curve indicates again, however, that it is too small.

If we accept these deviations as real it follows first that after the origin of this difference has been discovered, and it has been taken into consideration, the probable error of the measures would come out still smaller than is given above. But it is of much more importance that the discovery of the occurrence of this anomaly permits a further insight into the binary system under investigation, as I shall show below.

The fact that a periodic doubling of the lines is not perceptible might lead us to assume that one of the bodies of the system is dark, but the marked weakness of the lines of the spectrum of the clèveite gas leads us to conclude, however, that a second spectrum is superposed upon it. The broad and diffuse hydrogen lines, moreover, do not correspond to the typical spectrum of Class I*b* and lead to the further assumption that the superposed spec-

trum must belong to Class Ia2.[1] The very delicate metallic lines which appear in this class of spectrum along with the broad and diffuse hydrogen lines, disappear completely as the result of the superposition of this spectrum by the spectrum of Type Ib of the rapidly revolving body. On these assumptions it is now very easy to explain why the measures on $H\gamma$ yield smaller velocities than those on the cleveite lines. The narrower and less diffuse line of the Ib spectrum broadens and strengthens the maximum of absorption of the very broad and diffuse $H\gamma$ line of the Ia2 spectrum when the superposition is absolute. When the two spectra are relatively displaced to each other the line of the Ib spectrum remains within the broad line of the other spectrum, but the intensity curves of the two lines are so added together that there results a broad maximum, unsymmetrically placed with respect to the center of the combined image of the two lines. The measure of the $H\gamma$ line of the periodically displaced Ib spectrum is therefore affected by the $H\gamma$ line of the Ia2 spectrum, and this occurs in such a manner that the measures of the displacement, regarded absolutely, come out too small. The extent to which this occurs naturally depends wholly on the relative intensity of the absorption lines of the two spectra and on the quality of the spectrogram. In spite of the faintness and breadth of these absorption lines it is to be assumed that two maxima could be perceived in the compound $H\gamma$ line at the time of the greatest displacement if both components of the double star are strongly displaced. Even with a less motion of the second star the effect on the *Auffassung* of the compound $H\gamma$ line would have to be stronger than was actually the case. The assumption therefore seems admissible from the observations that the center of gravity of the two bodies must lie very close to the star with the Ia2 spectrum, or even within it.

After being led to these considerations by the observations I re-examined all the plates of the spectrum of *o Persei* and found

[1] The width of the hydrogen line $H\gamma$ is about 1.0 rev., that of the helium line 0.3 rev. One revolution of the screw of the measuring machine is 0.25 mm, and the displacement of this amount corresponds to a velocity of 290 km at $H\gamma$, or 340 km at λ 4472.

a very good confirmation of the suspicions above expressed. With a strong positive velocity the brightest point (on the negative) lay unsymmetrically to the center of the $H\gamma$ line, and the line was more diffuse on the side toward the violet, as was clearly seen on Plates 1213, 1234, 1245, 1265. With strong negative motion $H\gamma$ was more diffuse toward the red, distinctly on Plates 1224, 1252, and 1282, and not quite certainly on Plates 1257 and 1271. With the small motion the brightest point (on the negative) in the $H\gamma$ line was exactly symmetrical to the center of the line; the intensity of the absorption was distinctly greater than on plates at other times, particularly on Plates 1228, 1237, 1239, 1253, and 1262; also with a motion of —40 km on Plate 1242, the $H\gamma$ line appeared as on the above-mentioned plates.[1]

Taking 110 km as the greatest velocity, and assuming that the center of gravity of the system lies within the one body, and adopting the period 4.39 days, the distance of the two bodies is computed to be $\frac{6640000 \text{ km}}{\sin i}$, and the mass of the system to be $\frac{0.6 \odot}{\sin^3 i}$, where i is the angle included between the normal to the orbital plane to the system and the line of sight. I have taken for the greatest velocity the value obtained from the curve and also that directly observed. According to the considerations cited above, the mean values from three measured lines might be increased by about 5 per cent., which would increase the maximum values by about 5 or 6 km. If we base our computation upon a maximum velocity of 115 km instead of 110 km we obtain for the distance of the two bodies $\frac{6940000}{\sin i}$, and for the mass of the system $\frac{0.7 \odot}{\sin^3 i}$.

[1] I ought not to omit to mention that in my previous investigation of the motion of α *Virginis* I was led to similar considerations. *Pub. des Astrophys. Obs.*, **7**, 139.

THE MASS OF 85 *PEGASI.*

By George C. Comstock.

This is an interesting binary star ($\alpha = 23^h\ 56^m\!.8$; $\delta = +26°\ 33'$), discovered by Burnham in 1878, and soon recognized as possessing an unusually rapid orbital motion. Despite the large difference of magnitudes between its components, 5.7 and 11.3, and their small angular separation — the distance is always less than a second of arc — the star has been well observed, and fairly accordant orbits have been obtained for it by four different computers. The latest of these, by Burnham, assigns to the star a periodic time of revolution of 25.7 years and a semi-axis major of $0''\!.78$. There is a ninth-magnitude optical companion to the binary whose relative position was first measured by Otto Struve in 1851 and which has been well observed since that time, with the exception of one serious gap between 1852 and 1865, as shown in the following table.

I have recently discussed the comparisons of this star with 85 *Pegasi* to determine the relative proper motion of the stars and have been thus led to consider the possible motion of the bright component of 85 *Pegasi* with reference to the center of gravity of its system. The data of the problem are the observed position angles and distances, θ and ρ, of the ninth magnitude star, C, referred to the bright star, that I shall call A, and similar data, p and s, for the faint companion, B, of the binary system. These latter co-ordinates I have derived from Burnham's apparent orbit[1] for the dates of the available observations of C. I desire here to emphasize the great interest and value of these apparent orbits and to express the wish that a cut showing the apparent orbit might always be published in connection with orbit determinations of binary stars.

For the discussion of the observation we may put

$\mu =$ the difference of declination, at an assumed epoch, between C and the center of gravity of A and B.

[1] *Publications of the Yerkes Observatory*, 1, 270.

ν = the relative proper motion of C and the center of gravity above defined.

τ = the time interval between the given observation and the epoch 1885.54.

k = the fractional part of the distance A — B included between the center of gravity and the star A.

Transforming polar to rectangular co-ordinates, we find for the co-ordinate parallel to the hour circle,

$$\mu + \tau\nu + s \cos p \cdot k = \rho \cos \theta,$$

with a similar equation for the co-ordinate perpendicular to the hour circle,

$$\mu' + \tau\nu' + s \sin p \cdot k = \rho \sin \theta.$$

For the determination of the five unknowns in these equations I have employed the following observations, in which the posi-

OBSERVED CO-ORDINATES, A, C.

Date	Nights	θ	ρ	W	Observer
1851.96	1	114° 3′	33.03	0.5	OΣ
52.67	1	113 51	32.60	0.5	OΣ
65.91	1	92 9	18.89	0.5	OΣ
68.77	1	82 24	17.03	0.5	OΣ
70.00	44	77 1	16.06	2	Brünnow
74.66	1	54 24	13.92	0.5	OΣ
76.77	1	40 18	14.02	0.5	OΣ
78.54	4	33 36	14.40	1	β
78.74	1	32 48	14.76	0.5	Dembowski
79.27	8	30 24	14.96	1	β
80.57	4	25 0	15.41	1	β
81.54	4	20 48	16.29	1	β
81.88	1	19 48	16.54	0.5	Bigourdan
82.62	1	15 12	16.98	0.5	OΣ
82.77	3	17 6	17.34	1	β
83.54	1	11 18	17.34	0.5	Seagrave
86.24	3	7 36	19.84	1	HΣ
86.99	3	6 6	21.15	1	Englemann
88.67	5	0 54	21.71	1	β
89.50	4	358 42	22.66	1	β
89.82	2	358 24	22.70	1	Leavenworth
90.52	3	356 42	23.59	1	β
91.56	3	354 42	24.58	1	β
91.94	8	354 18	25.02	1	β
95.06	1	350 0	28.86	0.5	Lewis
95.69	3	348 42	29.27	1	Aitkin
96.75	2	347 48	30.48	1	Aitkin
97.56	2	346 6	31.49	1	Aitkin
97.82	2	345 42	31.74	1	β
98.49	3	344 24	32.53	1	β
98.69	2	344 27	32.90	1	Aitkin
1902.73	3	341 19	37.64	1	Comstock

tion angles, θ, are reduced to the equinox of 1850.0. Since the present discussion is a part of a larger investigation of the proper motions of faint stars, I have assigned to the several observations weights, W, in accordance with a uniform system adopted for that work. While these weights could probably be modified with some advantage, in the present case I do not think that any substantial change would be found in the resulting values of the unknowns.

Numerical equations of the type given above are so readily formed from these data that I refrain from publishing them and pass immediately to the following groups of elimination equations that result from their least square solution.

In Declination	In Right Ascension
$\mu - 0.0018\nu - 0.3215k = +18''.879$	$\mu' - 0.0018\nu' - 0.1175k = +2''.602$
$\nu - 0.0974k = +9.729$	$\nu' + 0.0706k = -8.307$
$k = +0.631$	$k = +0.604$

The time unit here employed is a decade, but the resulting values of ν and ν' printed below are expressed as annual variations.

The close agreement in the values of k furnished by these two groups of equations inspires some confidence as to its being a real quantity, and in confirmation of this view we may note that physically the value of k must fall between 0 and $+1$, as in fact it does result from the solutions, while a mere computation result is subject to no such restrictions and might have any value whatever.

I adopt as the definitive result of the investigation $k = +0.62$ and with this value find for the epoch 1885.54:

$$\mu = +19''.10 \qquad \mu' = +2''.66$$
$$\nu = +0.9789 \qquad \nu' = -0.8350$$

I have also derived values of these quantities corresponding to the supposition $k = 0$, and have formed the sums of the weighted squares of the residuals and the probable error of an equation of unit weight, corresponding to these two hypotheses, viz.:

$$\text{For } k = 0 \qquad [pvv] = 5''.44 \qquad r_1 = \pm 0''.20$$
$$k = 0.62 \qquad [pvv] = 3.04 \qquad r_1 = \pm 0.15$$

These results show a substantial improvement, due to the intro-

duction of an orbital motion for the star A, and corresponding to the adopted value of k I find for the ratio of the masses of the components of 85 *Pegasi*,

$$A : B = 3 : 5.$$

For the transition from these ratios to the absolute masses of the stars there are available three determinations of the parallax of 85 *Pegasi*, viz.:

Brünnow -	$\pi = +0''.05 \pm 0''.020$	Filar micrometer,
Flint - - -	$= +0.02 \pm 0.038$	Meridian transits,
Flint - -	$= +0.04 \pm 0.038$	Meridian transits,

and adopting $+0''.04$ as the mean of these values I obtain for the respective components, in terms of the Sun's mass taken as unity,

$$A = 4.3 \qquad B = 7.0,$$

but the adopted parallax is so small as to render these results decidedly uncertain.

The relative masses, however, seem to stand upon an altogether different footing as respects the accuracy with which they are determined, and in the light of current theories of stellar development they present a somewhat remarkable result: A star, A, whose spectrum is of the second type (E, in the *Draper Catalogue*) emits more than 100 times the light of its companion, B, although B is presumably of equal age with A and possesses 60 per cent. more mass than the latter star.

WASHBURN OBSERVATORY, MADISON, WIS.,
January 23, 1903.

THE NEW GASES NEON, ARGON, KRYPTON, AND XENON IN THE CHROMOSPHERE.

By S. A. Mitchell.

From a historical point of view, D_3 is one of the most important lines in the spectrum. When examining in April 1895 the spectrum of a specimen of clèveite, Ramsay announced the discovery of a substance that gave the characteristic helium line, it was felt that a great triumph had taken place for the methods of the "new astronomy." About the same time Rayleigh and Ramsay discovered another new element, argon, and in the early summer of 1898 Ramsay found two more elements, krypton and neon, and subsequently a heavier gas to which the name xenon was applied. Making use of the extremely low temperatures of liquid air and liquid hydrogen, it was found that these five new gases were present in atmospheric air. Investigations of the properties of these gases [1] seem to indicate that the atomic weights are: helium, 4; neon, 20; argon, 40; krypton, 82; xenon, 128; and that they form a series in the periodic table between that of fluorine and that of sodium.

The lines of helium are such prominent ones in the chromospheric spectrum that it would be interesting to see if the other new gases are also present in the chromosphere, and accordingly comparisons have been made between the spectra of the "flash" and of the new elements whose wave-lengths have lately been published.

The flash spectrum was photographed by the writer at the Sumatra eclipse,[2] with an apparatus consisting of a Rowland plane grating of 15,000 lines per inch, having a ruled surface of $3\frac{1}{2} \times 5$ inches, and a quartz lens, the whole mounted so as to give a normal spectrum. The photographs—like most of those at this eclipse—were made through clouds. These poor

[1] Ramsay and Travers, *Proc. R. S.*, **67**, 329, 1900.

[2] Astrophysical Journal, **15**, 97, 1902.

weather conditions, however, did not interfere with the spectrum as much as was expected, which is shown by the fact that 374 lines were measured between F and H. The dispersion employed was about one-fifth that of the largest Rowland concave gratings of 21½ feet radius, and 20,000 lines per inch, and about equal to the dispersion of the Lick and Yerkes spectrographs.

Comparisons of the *intensities* of the lines, and of the *numbers* of the lines due to the different elements in the flash and in the solar spectrum, as given by Rowland, led to the division of the elements into three groups (*loc. cit.*, Table V): those giving (1) lines strong in the flash and strong in the solar spectrum; (2) lines strong in the flash, weak in the solar spectrum; and (3) lines weak in the flash, but strong in the solar spectrum.

To the second group belong *H, He, Sc, Ti, V, Cr, Mn, Sr, Y*, and *Zr*. It has been shown[1] that helium, in consequence of its small density, ascends to great heights above the Sun's surface, and as its layer is covered up gradually by the Moon at the time of an eclipse, the resulting exposure is many times that given to denser but shallower layers, and consequently helium lines in the flash spectrum are very prominent. Taking into account the increasing atomic weights of the series of new gases, and also the behavior of the light and heavy vapors in the Sun's atmosphere, as found out by investigations of the flash, we should expect, as in the case of helium, none of these gases to be found in the ordinary solar spectrum. We should also expect lines of the more volatile gases of the Earth's atmosphere, neon and argon, of atomic weights 20 and 40 respectively, to be present in the flash, while those of the less volatile gases, krypton and xenon, of atomic weights 82 and 128 respectively, are most probably not to be found there.

No lines of these gases appear in the ordinary solar spectrum, but if we make a detailed comparison of their spectra with that of the flash, there seem to be certain lines of the latter that are undoubtedly due to these new gases of the atmosphere.

The most volatile of these were obtained[2] from their solution in liquid air by fractional distillation at low pressure, in

[1] *Ibid.*, p. 117.

[2] RAMSAY and TRAVERS, *Proc. R. S.*, **67**, 329, 1900.

this way removing the greater portion of the helium and neon from this mixture of gases, leaving the argon behind. Many attempts were made to separate the helium from the neon, which were not successful until these gases were subjected to the temperature of liquid hydrogen, when neon was liquified and perhaps solidified, while the helium remained gaseous.

The more volatile gases of atmospheric air uncondensed at this temperature have been examined spectroscopically by Liveing and Dewar, and wave-lengths have been published.[1] In this spectrum appear lines due to neon, to helium, and to free hydrogen in the Earth's atmosphere. These wave-lengths, given, however, only to the nearest Ångström unit, have been compared with those of the lines of the flash spectrum.[2] Although it is difficult to identify with certainty when wave-lengths have no greater accuracy than this, it seems highly probable that nearly all the stronger lines of the most volatile of the new gases are found in the flash, some of them agreeing with lines already identified as corresponding to Fraunhofer lines (in this case it being impossible to separate the lines), other lines identified with lines that have no counterpart in the ordinary solar spectrum. None of the lines of the flash due to these more volatile gases are strong lines like those of helium, their intensities being 0 on a scale where 0 denotes a line seen with certainty and 10 is the intensity of the strongest line. Lines in the flash which seem to belong to these gases and to no others are those at $\lambda\lambda$4047, 4398, 4422, 4431, 4540, and 4844.

Several tables of wave-lengths of argon have been published. Kayser employed a large concave grating and published[3] his results to thousandths of a tenth-meter. Most of the strong lines of argon are found in the flash spectrum — but again as weak lines only. Argon lines appear at $\lambda\lambda$4180.3, 4200.8, 4259.5, 4266.8, and 4430.3.

The most accurate wave-lengths of krypton are those of Runge, measured with a concave grating and given[4] to thousandths of a tenth-meter. The strongest lines are in a part of

[1] *Ibid.*, p. 467.

[2] ASTROPHYSICAL JOURNAL, **15**, 103, 1902.

[3] *Ibid.*, **4**, 1, 1896.

[4] *Ibid.*, **10**, 73, 1899.

the spectrum not covered by the flash photographs, and there seem to be no krypton lines in the flash.

The only wave-lengths of xenon are those of Liveing and Dewar,[1] given only to the nearest tenth-meter. Some of the strongest of the xenon lines do not appear in the flash, while the wave-lengths of some less strong seem to agree with those of flash lines; but as the wave-lengths are not accurate enough, it is impossible to say more than that the presence of xenon in the Sun's atmosphere is doubtful.

Consequently, it seems that the more volatile gases of atmospheric air uncondensed at the temperature of liquid hydrogen, together with hydrogen, helium, neon, and argon, are present in the chromosphere, while the evidence in regard to krypton and xenon is inconclusive.

The finding of these gases in the Sun and the undoubted presence of free hydrogen in the Earth's atmosphere have an importance for cosmical physics that can hardly be overestimated. According to Liveing and Dewar, "if the Earth cannot retain hydrogen nor originate it, then there must be a continued accession of hydrogen to the atmosphere (from interstellar space), and we can hardly resist the conclusion that a similar transfer of other gases must also take place,"[2] as has been shown by these distinguished physicists, and again by Dewar in his presidential address before the British Association for the Advancement of Science, these new gases, and particularly the more volatile gases of atmospheric air, play an important part in the spectra of the aurora, of nebulæ, and of the corona. Of more than a hundred auroral rays observed by Stassano, more than two-thirds appear to belong to the more volatile gases of atmospheric air, while the majority of the remainder seem to belong to argon, krypton, and xenon. We are also told by Dewar that of a "list of 339 lines photographed by Humphreys, during totality" (this, however, was called the spectrum of the corona, whereas it was the spectrum of the chromosphere), "only 55 do not differ by more than one unit on Ångström's scale from lines measured in the most volatile gases of the atmosphere, or in

[1] *Proc. R. S.*, **68**, 389, 1901. [2] *Ibid.*, **67**, 468, 1900.

krypton or xenon." It seems rather to the present writer that the great majority of these lines more closely correspond to Fraunhofer lines than to the lines of these rare gases.

These gases may take their origin from the Earth itself; in fact, helium and neon are occluded from the waters of the Bath spring in England. The presence of free hydrogen in the atmosphere cannot be explained in this way. It is more likely that hydrogen comes to us in small ionized particles from the Sun, being sent hither, as has been shown by Arrhenius,[1] by the pressure of light, and likewise helium and the more volatile gases are present in the atmosphere through being repulsed from the Sun by the ionization of small particles of these gases.

It seems therefore that the finding of these new gases in the Sun's chromosphere is an independent verification of the truth of the theory of Arrhenius, which states that particles of matter are being continually scattered throughout the universe, starting from one sun and reaching another, with the result that all bodies of the universe are gradually becoming more and more alike.

COLUMBIA UNIVERSITY,
New York City, February 1903.

[1] *Physikalische Zeitschrift*, November 1900; see also COX, *Popular Science Monthly*, January 1902.

ON THE OCCURRENCE OF SPARK LINES IN ARC SPECTRA.[1]

By J. HARTMANN and G. EBERHARD.

IN November of last year Dr. H. Konen published in the *Annalen der Physik* (**9**, 742 ff., 1902), some very valuable researches which he had made on the spectra of electrical discharges in water. This induces us to communicate briefly in what follows the results of investigations in the same field which we carried out last autumn.

In the first place it should be remarked that our observations confirm those of Konen even to details. We have, however, observed some additional phenomena not described by Konen, which seem to be of importance in the interpretation of the processes of electrical luminosity.

Our experiments at first were confined to a thorough study of the spectra of magnesium and silicon, which are of special importance in astrophysics. While we were varying as much as possible the conditions under which these elements were made luminous, we made use, among other things, of the electric arc under water. Here we noticed with surprise that in the spectrum of the arc there appeared lines which had hitherto been regarded as peculiarly characteristic of the spark spectrum.

Konen also has shown for a number of metals that lines in the spectrum of the metallic arc under water are not superposed on a strong continuous spectrum like the spark spectrum produced under the same conditions, but appear only as bright and generally sharp lines; the appearance in the arc under water of lines characteristic of the spark spectrum, however, has not to our knowledge been known hitherto.

When the arc is passed under water between two electrodes of metallic silicon, the spark lines λ 4128, and λ 4131 appear with almost the same intensity as does the chief arc line λ 3905. Under the same experimental conditions magnesium shows the

[1] Translated from advance proofs, furnished by the authors, of a paper to appear in the *Sitzungsberichte der K. Akad. zu Berlin.*

line at λ4481, hitherto regarded as characteristic of the spark spectrum, and indeed it makes almost the strongest line in the whole spectrum; at any rate, its intensity considerably exceeds the arc line of magnesium at λ4352. Both elements further exhibit the noteworthy phenomenon that the lines mentioned, which are known only as very broad and weak lines in the spark discharge in air, are considerably narrower and become more sharply defined, despite their intensity, when produced by the arc under water.

In order to ascertain whether the phenomenon described above was general, the experiments were also extended to other metals, and we were able to produce results entirely analogous with zinc and cadmium. With zinc the very diffused spark lines λ4911 and λ4924, when produced by the arc under water appeared strong and relatively sharp. The lines λ5339 and λ5379, which Kayser and Runge failed to find in the arc spectrum of cadmium, together with λ4416, have also been easily obtained by us with the arc under water. We found the aluminium lines λ4513 and λ4530 less strong.

Carbon alone remained essentially anomalous, giving, with the arc under water, as Konen also found, no lines, but a band spectrum only.

We now suspected, in connection with a remark by Schenck[1] that the temperature of the electrodes and metallic vapors, no doubt much reduced by the water, might have an influence on the nature of the spectrum, and we thereupon greatly cooled zinc electrodes with liquid air, without obtaining, however, any material change in the arc spectrum. We then allowed sparks from an iron electrode to pass over to zinc placed in a crucible. While the zinc was heating to the melting-point and beyond, several observers (Vogel, Müller, Kempf, Hartmann, Eberhard) estimated the relative intensity of the pair of lines at λ4911 and λ4924 as compared with the three lines at λ4680, 4722, 4809, and it appeared in every case that the intensity of these three lines was greatly augmented in comparison with the pair at λ4911 and λ4924 as soon as the zinc was heated.

[1] Astrophysical Journal **14**, 131.

In the success of this experiment we might see a confirmation of the above-mentioned conjecture that with ascending temperature of the electrodes the spectrum of the spark approached that of the arc. But it is not improbable that the observed phenomenon occurred merely through the increase of the metallic vapor due to heating the zinc, whereby, on account of the increase of these vapors, the resistance in the path of the spark became less and the spark lines accordingly gave place to the arc lines.

Since an answer to the question entirely free from objection cannot therefore be obtained by these means, we have endeavored to find an explanation in another direction. From former researches (Crew, Basquin) it is known that an atmosphere of hydrogen influences the arc spectrum of magnesium in such a manner that the spark line λ 4481 stands out prominently. Accordingly we have photographed the arc spectra of a series of metals in a current of hydrogen. It now appeared that spectra obtained in this manner are in fact almost identical with the arc spectra under water, and we are therefore of the opinion that hydrogen, released by electrolysis around the electrodes in water, causes the transformation of the arc spectrum into the form observed by us.

It should be observed that all lines mentioned by us in the foregoing are diffuse and hazy in the spark spectrum, and that it is also just these lines which can be made to disappear in the spark spectrum by the introduction of suitable self-induction, as we have convinced ourselves by our own experiments.

It follows from our investigations that it is not permissible to set apart individual lines of the spectra of metals as characteristic of the spark or of the arc respectively, and from their appearance to draw conclusions regarding the temperature of the corresponding processes of luminosity. The latter point applies particularly in the interpretation of stellar spectra, for which some have sought to make positive statements as to the temperature of stellar atmospheres, based upon the behavior of individual magnesium and silicon lines.

POTSDAM.
February 1903.

THE POSITION OF RADIUM IN THE PERIODIC SERIES ACCORDING TO ITS SPECTRUM.

By C. Runge and J. Precht.

The spark spectrum of radium can be admirably observed with the radium bromide recently produced by Mr. Giesel. A few milligrams, which Mr. Giesel was kind enough to place at our disposal for this purpose, was sufficient with a low dispersion for obtaining the spectrum in a much more complete manner than has hitherto been observed, and with greater disperson the readily photographable lines could be investigated as to their behavior in a magnetic field. We thus found that the strongest radium lines are precisely analogous to the strongest lines of barium and the corresponding lines of the related elements *Mg*, *Ca*, and *Sr*. As has been shown by Runge and Paschen,[1] these lines can be grouped as three pairs, which, on account of certain analogies with the spectra of the alkalies, may be designated as the pair of the principal series, the pair of the first subordinate, and the pair of the second subordinate series. In the case of the pair of the first subordinate series there appears along with the line of the greater wave-lengths a fainter line on the side of longer wave-lengths, which Runge and Paschen designate as a satellite. The two lines of each of these three pairs have the same distance for each element, reckoned on the scale of vibration numbers, except that for the pair of the first subordinate series the satellite must be taken instead of one of the lines. But from element to element the distance is, on the contrary, different, increasing in a very regular way with increasing atomic weight, as will be discussed more particularly below. In the magnetic field these lines are resolved into components in a different way, as Runge and Paschen have shown, but so that, reckoned on the scale of vibration numbers, the separation of each line of one element is exactly the same as the separation of the *corresponding* line of each of the other elements.

[1] Astrophysical Journal, **16**, 123, 1902.

We have now found that precisely the same thing holds good for radium, so that *Ra* is to be placed with *Mg*, *Ca*, *Sr*, and *Ba* in a group of chemically related elements, as is also demanded by the chemical behavior of radium, in so far as this is known.

The corresponding lines are collected together in the following table:

	Mg	*Ca*	*Sr*	*Ba*	*Ra*
Principal series	2803 2796	3969 3934	4216 4078	4934 4554	4682 3815
First subordinate series	.... 2798 2791	3181 3179 3159	3475 3465 3381	4166 4131 3892	4436 4341 3650
Second subordinate series	2937 2929	3737 3706	4306 4162	4900 4525	5814 4533

The separation of the radium line in the magnetic field is very well observable in case of the strongest lines; as yet we have been unable to resolve only the satellite and the line in the green at λ 5814.

As remarked above, the distances between the two lines of the pairs are the same for every element on the scale of vibration numbers. This is also true for radium, as is shown in the following table:

	λ	$\frac{10^8}{\lambda}$	Distance
Principal series	4682.35 3814.59	21356.8 26215.1	4858.3
First subordinate series	4436.45 3649.77	22540.5 27399.0	4858.5
Second subordinate series	5813.9 4533.33	17200.2 22058.8	4858.6

The deviations of the three distances from each other are sufficiently explained by the errors of observation. They correspond to very small errors in determinations of wave-length.

For *Mg*, *Ca*, *Sr*, and *Ba* the distances increase with the atomic weight from element to element:

	Atomic Weight	Distance
Mg	24.36	91.7
Ca	40.1	223
Sr	87.6	801
Ba	137.4	1691

It is natural to regard the atomic weight as a function of the distance and to extrapolate this function for radium. It has already been pointed out by Rydberg and by Kayser and Runge in their papers on "The Spectra of the Elements" that the distances of the pairs of lines within a group of chemically related elements increase in a regular way with the atomic weight. They state for the alkalies that the atomic weight is very nearly proportional to the square root of the distance. We would call attention to the fact that for the other group in which pairs of lines have been observed, the relationship between of the pair and the atomic weight may be represented by a simple formula, viz.:

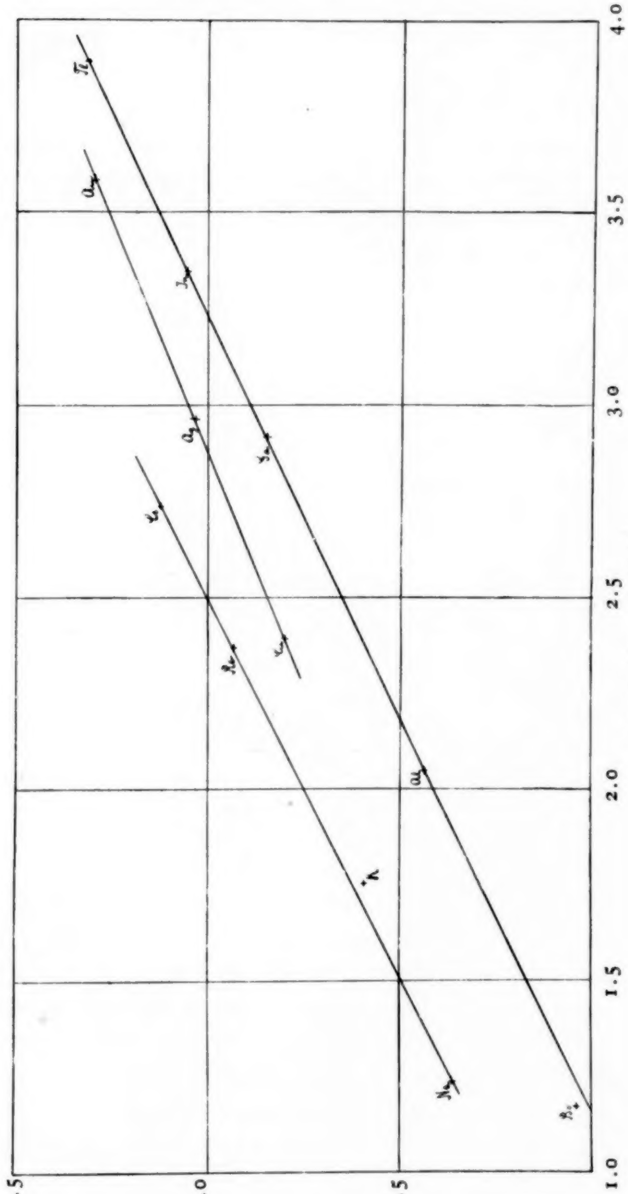
In every group of chemically related elements the atomic weight is proportional to some power of the distance of the two lines of the pairs.

In general the exponent is not a whole number.

Expressed in other words this formula becomes:

If the logarithms of the atomic weight and distance be taken as co-ordinates, the corresponding points of a group of chemically related elements will lie in a straight line.

This law is shown in the two following figures. We see from Fig. 1 that among the alkalies, potassium is the only one which falls below the straight line through the remaining points. We would not here assert that the observed atomic weight of potassium is incorrect; but it seems to us interesting that this law of the straight line exhibits a larger deviation for just that element which is anomalous in respect to the periodic series, in so far as the atomic weight of *K* would have to be greater than that of argon in order to fit into the periodic series. As regards boron, gallium, and indium, the pairs of lines have not been investigated in a magnetic field. We cannot, however, doubt the



Logarithms of Atomic Weight.

Logarithms of Width.

(0.01 division of scale = 2.3 per cent.)

FIG. 1.

existence of the corresponding pairs. The same is true of the alkalies, where only the yellow lines of sodium have been investigated in a magnetic field.

Fig. 2 represents the same relationship for *Mg*, *Ca*, *Sr*, *Ba*, and *Ra*. Extrapolation gives 258 for the atomic weight of radium. We can, of course, somewhat displace and rotate the straight line without removing it too far from the points, but the figure clearly shows that the value 225 determined by Madame Curie is decidedly off the line.

In the following table the straight line is replaced by a formula and the extrapolation by computation. If x denote the separation of the pair on the scale of vibration numbers, $\times \frac{10^8}{\lambda}$, we have:

$$\text{Atomic weight} = \text{No. whose log is } (0.2005 + 0.5997 \log x.)$$

	Atomic Weight	
	By Formula	Observed
Mg	23.84	24.36
Ca	40.6	40.1
Sr	87.5	87.6
Ba	136.9	137.4

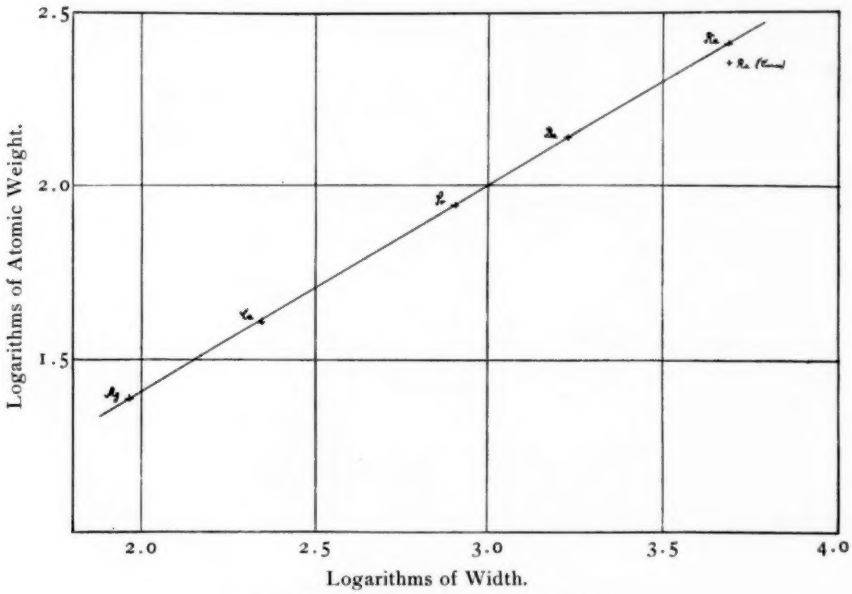
Extrapolation for radium gives:

Computed atomic weight of radium = 257.8.

We cannot risk the assertion that our figure deserves more confidence than the value determined by Madame Curie, but it should nevertheless be said that, in view of the close relationship between barium and radium, and of the small quantities of the substance with which the chemists are obliged to work, the complete separation of these two substances is very difficult; and further, that with an imperfect separation Madame Curie would necessarily obtain too small an atomic weight.

According to crystallographic observations soon to be published by F. Rinne, the bromides of radium and barium are isomorphous with each other, so that it is very probable that the two substances would crystallize together (in an isomorphous

mixture), similar relations as to solubility being assumed. The great difficulty of separating the two substances from each other by a process of crystallization is due to this, and it may happen that even with frequently repeated crystallization greater or less quantities of barium may be present in the corresponding radium compound. The number 223 fits the periodic system better in the respect that it fills the gap between bismuth and thorium in



(0.01 division of scale = 2.3 per cent.)

FIG. 2.

the correct column. For atomic weight 258 radium would have to be pushed two rows farther along in the column *Mg*, *Ca*, *Sr*, and *Ba*, and a number of new unoccupied places would arise in the periodic system.

On the other hand, we may adduce for the higher atomic weight the point remarked by Rutherford. The greater atomic weight suggests a complicated structure of the atom and hence a more easy breaking up into electrons. The element which most strongly emits electrons should therefore also have the greatest atomic weight.

The radium line at $\lambda 4826.14$, which is the most conspicuous in the Bunsen flame, is in respect to the separation in the magnetic field analogous to the strongest line in the Bunsen flame, *Ba* $\lambda 5535$, *Sr* $\lambda 4607$, *Ca* $\lambda 4246$. All these lines separate into three components which for all these elements are equally separated from each other on the scale of vibration numbers.

PHYSIKALISCHES INSTITUT DER TECHNISCHEN HOCHSCHULE,
Hannover, Germany, January 1903.

NOTE ON SOME EFFECTS OF RULING ERRORS IN GRATING SPECTRA.

By A. S. King.

The possibility that errors of ruling in diffraction gratings may produce "ghosts" near strong lines in the spectrum makes it desirable that we understand as fully as possible the forms that such ghosts may take. If they occur in the most frequent form, as two lines symmetrically placed with respect to the primary line and close to this line, they are readily recognized. The writer has observed, however, that a very good grating may give, in certain regions of the spectrum, ghosts of higher order at such distances from the primary line that they might easily be mistaken for real lines.

The theory of ruling errors is a complex one. The most complete treatment we have is that given by Rowland in his article entitled "Gratings in Theory and Practice."[1] In this Rowland establishes general relations as to position and intensity of ghosts, and develops in some detail the expressions for intensity of ghosts of different orders resulting from small periodic errors of ruling. He also calls attention to other effects which may result from special conditions.

The fundamental formulæ give the position of the ghost of nth order by the relation $\mu_n = 2\pi N/ba_0 \pm ne_1/ba_0$, and the intensity by the function $J_n^2(b\mu_n a_1)$, where the first term of μ_n gives the position of the primary line, and the second term the position of the ghost with respect to the primary line. N denotes the order of spectrum, a_0 the grating space, a_1 and e_1 the amplitude and period of ruling error, and $b = 2\pi/\lambda$. In accordance with these relations, we have the following characteristics of ghosts:

1. Ghosts of the same order are at equal distances on each side of the primary line, and of equal intensity.

2. The distance of a ghost from the primary line varies

[1] *Astronomy and Astro-Physics*, **12**, 129, 1893.

directly with the wave-length, but is *independent of the order of the spectrum*, since the second term in the value of μ_n does not contain N.

3. The ghosts of different orders are equally spaced.

4. The intensity increases rapidly with the order of the spectrum and with the amount of ruling error.

5. The intensity of the primary line is diminished by an amount equal to the sum of intensities of the ghosts.

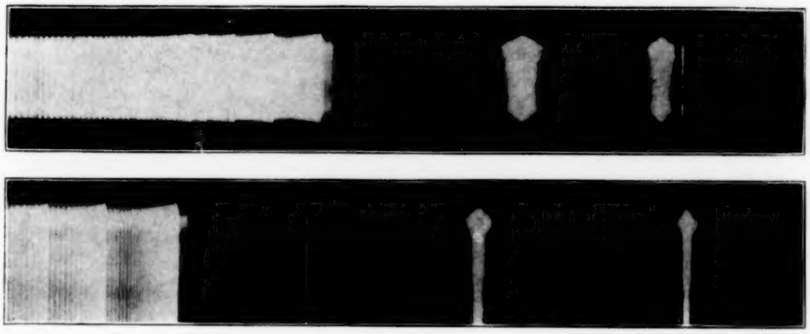
6. Since, for the same order of spectrum, the relative intensity of any ghost depends upon the value of a_1 in the expression $J_n^2(b\mu_n a_1)$, we may have the greatest intensity in a ghost of higher order than the first.

7. As $\mu = \sin\alpha$, these relations are constant only for a given position of the grating, and we have a new set of values when the angle of incidence is changed, as when we move the grating in order to observe another part of the spectrum.

The writer has recently obtained some photographs which show very clearly the effects of the last two determining elements given above. The concave grating used was one of 15 feet radius and 15,000 lines to the inch. In the first-order spectrum a prolonged exposure is required to show any ghosts on the photographic plate. On such over-exposed plates a faint appearance resembling the head of a band is seen at wave-length λ3888.75, which the writer tried in vain to identify with any known element of the spectrum, and which proves to be a first-order ghost of the λ3883 cyanogen band. Faint ghosts also appear beside very strong metallic lines, as the strontium line at λ4215.

In the second- and third-order spectra, however, remarkable series of ghosts appear above each of the cyanogen bands at λ3590, 3883, and 4216, and also near strong metallic lines in that region, as the H and K lines and the strontium line above mentioned. In each of these cases we have very distinct ghosts up to the fourth order, giving above each of the cyanogen bands the appearance of a second band with four equally spaced heads. Furthermore, the intensity of these ghosts does not diminish uniformly as they recede from the primary line. Those belonging to the head of the 3883 band occur at λ3888.75,

PLATE XVI.



GHOSTS IN GRATING SPECTRA.

3893.90, 3899.06, 3904.22, and have intensities in about the ratio 10, 5, 2, and 8. The ghost of fourth order is thus almost as strong as that of first order. The heads of the bands at $\lambda 3590$ and $\lambda 4216$ have ghosts of similar appearance, the spacing in each case being proportional to the wave-length. Photographs of these bands in the third-order spectrum show the ghosts much stronger than in the second order, but each series preserves the same relative intensities among the ghosts of different orders, and the spacing of corresponding series of ghosts in the two spectra is exactly the same; this last fact proving conclusively that they are ghosts.

The plate shows the second- and third-order spectra in the region of wave-length $\lambda 3900$, giving the head of the 3883 cyanogen band and the H and K lines of calcium, with their attendant ghosts. The ghosts of the H and K lines are faint in the reproduction, the ghost of third order being scarcely visible, but the fourth-order ghosts of the K line, reversed like the primary line, can be seen at wave-length 3926.81 and 3940.85. A remarkable feature is that the fourth-order ghost of the K line below the primary line is considerably stronger than the corresponding ghost above the line. This difference shows very distinctly in the negative, and is not in accordance with the theory of equal intensity in ghosts of the same order. Whether such a difference also exists in the ghosts of the cyanogen bands cannot be determined on account of the dense structure below the head. The plate shows the equal spacing of ghosts and the unequal dispersion of true lines in the two orders of spectra.

The writer has been unable to obtain ghosts of higher order than the first on plates taken in the region of wave-length greater than 4600, which shows that a change in the angle of incidence may produce important changes in the character of the ghosts. Prolonged exposures in the upper part of the second- and third-order spectra give first-order ghosts of considerable intensity near all strong metallic lines and carbon bands, but no ghosts of higher order can be distinguished.

The simplest method of detecting ghosts, and perhaps the only really decisive test is to photograph the suspected lines in

spectra of different orders. If they are ghosts, the spacing will be the same in the two orders; while real spectral lines will suffer a dispersion proportional to the order of spectra used.

The theory of ruling errors as given by Rowland explains the action of only the principal causes of ghosts. Other causes, as periodic variation in the depth of ruling, periodic waves in the surface, and perhaps changes in the shape of the groove made by the ruling point, may all give effects whose exact nature is not easily determined. For example, the variation of intensity among the ghosts of different orders shown on my plates does not agree with any case in Rowland's table giving the distribution of intensity among the ghosts according to the amount of ruling error. It is probable that other causes have a part in producing the effects observed. However, the general effect is predicted by Rowland's theory, and the results show that ghosts need not be of any fixed type, nor of the same appearance throughout the spectrum. The fact that the grating may give ghosts of higher order at a considerable distance from the primary line (7.02 tenth-meters in the case of the K line) and more intense than those nearer the line, shows the need of care, especially in working with spectra of second or third order, that such widely separated ghosts are not mistaken for actual spectral lines. Photographs taken in spectra of different orders will at once decide this point.

University of California,
February 1903.

ε *AURIGAE* A SPECTROSCOPIC BINARY.[1]

By H. C. Vogel.

In his investigations made a few years ago on the more refrangible parts of stellar spectra, Dr. Eberhard was struck by the fact that in the spectrum of the well-known variable ε *Aurigae*, which lies in the transition between types I and II, the series of hydrogen lines clearly extended farther beyond the H and K lines than is the case with stars of similar type. He suspected that the spectrum of the star should be regarded as the superposition of two spectra of different types.

Changes in the spectrum great enough to be recognized with the slight dispersion of the single prism spectrograph (*D*) used by Dr. Eberhard were not exhibited by plates taken at various times.

In the latter part of April and beginning of May, 1900, three spectrograms were obtained by Professor Hartmann with the large spectrograph (III) in connection with the 80cm refractor, but a comparison of these plates in the region from λ415 to λ455 showed nothing striking. On November 9, 1901, and November 18, 19, and 22, 1902, Dr. Eberhard further photographed the spectrum of ε *Aurigae* with the three-prism spectrograph (IV) designed by me three years ago for the photographic refractor of 32.5 cm aperture. A superficial comparison of the spectra taken in the latter year with those of the preceding year was sufficient to show that the spectrum had undergone a change. The thorough examination and measurement of the spectrograms which I at once began has so far furnished the result that the suspicions of Dr. Eberhard were well founded, and that the spectrum of ε *Aurigae* is in fact the result of the superposition of two spectra, the one similar to that of *α Cygni*, and the other lying between the first and second types, like *α Persei* or *γ Cygni*.

At present the first-named spectrum is the more intense and

[1] Translated from advance proofs, furnished by the author, of a paper communicated to the *Kgl. Akademie der Wiss. zu Berlin.*

it is displaced relatively to the other toward the violet by an amount which would correspond to a velocity of from 30 to 40 km per second. The spectrum is now distinguished from that of the previous year principally by the fact that but few lines of the iron spectrum are recognizable in it. Most of the lines have disappeared, probably as a consequence of the relative displacement of the spectra, and practically the only lines recognizable are those of the spectrum similar to *α Cygni*. Most of these appear double and are characterized by the fact that the component lying toward the violet is with few exceptions the stronger, and the boundary on the violet side of the double lines, which are often difficult to separate, is extremely sharp. This is particularly striking in case of the hydrogen lines, as is shown by a very successful plate taken by Professor Hartmann at my request on November 22, 1902, with the one-prism spectrograph (I) in connection with the 80 cm refractor.

There can accordingly be no doubt that ε Aurigae is a spectroscopic binary and probably one of very long period.

Considerable difficulties have been encountered in the comparison and measurements of the spectra because of the complications which result from dissimilarity of the superposed spectra, particularly in certain parts of the spectrum. I intend to communicate later more fully the very interesting details as to the spectrum of this star, which will be regularly observed here.

POTSDAM,
December 1902.

Minor Contributions and Notes.

TRANSPARENCY OF COMET 1902 *b*.[1]

THE statement is frequently made that comets are perfectly transparent, even faint stars being visible through them. The observations on which this statement is based appear to be very vague, as, even if careful comparisons were made, large errors might be introduced by the effect of the bright background formed by the light of the comet. The rapid motion of Comet 1902 *b* caused it to cover a large area, and therefore rendered it easier to find a star over which it would pass. After waiting for a suitable occasion, the observations given in Table I

TABLE I.

OBSERVATIONS.

G. M. T.	Difference	Residuals	Distance
h. m.			′
13 22.5	1.06	+.01	2.0
13 33.3	1.03	+.04	1.1
13 44.7	1.10	−.03	2.0
13 57.7	1.07	.00	4.0
14 10.8	1.06	+.01	5.5
14 26.7	1.06	+.01	7.9
14 46.9	1.08	−.01	11.0
15 2.3	1.12	−.05	13.1

were made by Professor O. C. Wendell with the polarizing photometer attached to the 15-inch equatorial. On the evening of October 14, 1902, the comet passed within about 1′ of the star $+21^{\circ}3483$, photometric magnitude 7.12. This star was compared with $+21^{\circ}3484$, magnitude 8.19. Each set of observations was the mean of sixteen settings. The Greenwich Mean Time is given in the first column, the difference in magnitude of the two stars in the second, and the deviation of this magnitude from the mean value, 1.07, in the third column. A positive sign indicates that the star $+21^{\circ}3483$ was faint, a negative sign that it was bright. The fourth column gives the distance of the nucleus of the comet from the star. The diameter of the coma was

[1] From *Harvard College Observatory Circular* No. 68.

about five or six minutes. The star was, therefore, covered by it in the first three observations. The nucleus resembled a star of about the tenth magnitude and the brightness of the coma was about that of a star of the ninth magnitude when spread over a circle one minute in diameter. The largest residual -0.05 is the last one, when the altitude of the comet was only 22°. The mean of all the residuals is ± 0.02. It appears, therefore, that the absorption of the light by the comet, if any, is insensible, and probably does not exceed one or two hundredths of a magnitude.

EDWARD C. PICKERING.

ADDITIONAL STARS OF THE *ORION* TYPE WHOSE RADIAL VELOCITIES VARY.

SPECTROGRAMS taken with the Bruce spectrograph since our communication in the March number of this JOURNAL show that the radial velocities of τ *Tauri* and ψ *Orionis* vary through a wide range.

In order to reach fainter spectra than can be photographed with the use of the cameras A and B, we have lately been experimenting with lenses of shorter focus (8 to 12 in. = 20 to 30 cm), with encouraging results. Several of the best types of modern anastigmat lenses have been loaned to us for trial by their manufacturers or agents. These show a great gain in the reduction of the exposure time, and it seems likely that with a camera lens of about 10.5 inches (267 mm) focus, especially constructed for our purpose, this result may be obtained without a greater loss of accuracy in the settings than would be justified under some circumstances. Pending the procurement of such a lens, the Bausch & Lomb Optical Co., of Rochester, N. Y., has most kindly allowed us to retain the "Unar" lens, Series I*b*, No. 7, which proved the most satisfactory for our purposes of all these lenses tried in connection with the spectrograph.

With this camera lens we have obtained plates of τ *Tauri* ($\alpha = 4^h\ 36^m$; $\delta = +22^\circ\ 46'$; Magnitude, *H. P.*, $= 4.4$) on three dates: 1903, February 25, 26, and March 5. The first plate gave a radial velocity of about $+70$ km, which, in view of the usually small values for stars of the *Orion* type, was strongly suggestive of a variation. The other two plates confirm this, and show a total range of about 75 km.

Three plates have also been obtained of ψ *Orionis* ($\alpha = 5^h\ 22^m$; $\delta = +3^\circ\ 1'$; Magnitude, *H. P.*, $= 4.7$), the first with the short focus

camera and the other two with camera A. The examination of the first A plate indicated a high positive velocity, which led to the measurement of the short focus plate previously obtained, on which accordant values were deduced from different lines, although the C plate should necessarily be given less weight than the A plate.

The data so far secured follow:

Plate	Date	Taken by	Velocity	No. of lines	Measured by
C 17	1903, February 4	F.	−122	3	A.
A 406	February 18	F.	+148	4	F.
A 415	March 12	F.	− 31	2	F.

It would seem probable that this star will show a very large total range when observations have been secured of its maximum and minimum radial velocities. Plates of other stars of the *Orion* type taken with the short-focus camera suggest variation, but we are not at this time prepared to report as to them.

Several plates have been obtained with the longer cameras of the star *η Hydrae* (also of Type I*b*) which cause us to have suspicions as to the constancy of its radial velocity. The spectrum is very difficult of measurement, owing to the diffuseness of the lines, and we shall follow it further.

EDWIN B. FROST and WALTER S. ADAMS.

YERKES OBSERVATORY,
March 16, 1903.

HENRY A. ROWLAND MEMORIAL LIBRARY.

IN order to establish a permanent memorial of the late Professor Rowland, and to promote the efficiency of the Physical Laboratory of the Johns Hopkins University, one of its former students has given a most generous sum of money with which to found a special collection of books, pamphlets, and other publications in the field of radiation and spectroscopy. This is to be called the "Henry A. Rowland Memorial Library," and is to be placed in the Physical Laboratory of the University. Mrs. Rowland has given the library all those books and pamphlets belonging to Professor Rowland, which refer to spectroscopy.

To make the collection complete, and to maintain its usefulness, the co-operation of observatories, laboratories, and investigators is necessary. It is earnestly hoped that sets of official publications,

books, reprints of papers on spectroscopy or allied subjects, and photographs of spectra and of apparatus will be contributed to the library, both now and in the future.

They may be addressed to the care of

JOSEPH S. AMES,
Director of the Physical Laboratory,
Johns Hopkins University,
Baltimore, Maryland, U. S. A.

PLATE XVII.

MARS.

September 2, 1894. $\lambda = 100°$.
36-inch refractor of Lick Observatory.